PARTIAL INTERACTION IN COMPOSITE STEEL AND CONCRETE BEAMS WITH MECHANICAL SHEAR CONNECTORS

by

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MECHANICAL SHEAR CONNECTORS

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ABSTRACT. This report presents a partial interaction analysis of composite beams with mechanical shear connectors. Relationships are developed between the degree of interaction and the degree of shear connection, and the slip-strain, slip and the distance between neutral axes.
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NOMENCLATURE

$A_c, A_s$  - cross-sectional area of the concrete and steel element respectively

$D$  - depth of composite beam

$d_s$  - diameter of shank of stud shear connector

$(AE)_c$  - axial rigidity of concrete element, $E_c A_c$

$(AE)_s$  - axial rigidity of steel element, $E_s A_s$

$(AE)_{cmp}$  - axial rigidity of the composite concrete and steel elements

where $(AE)_{cmp} = \frac{1}{(AE)_c} + \frac{1}{(AE)_s}$

$(EI)_c$  - flexural rigidity of concrete element, $E_c I_c$

$(EI)_s$  - flexural rigidity of steel element, $E_s I_s$

$(EI)_{cmp}$  - flexural rigidity of the composite concrete and steel elements

where $(EI)_{cmp} = (EI)_c + (EI)_s$

$\phi(x)$  - curvature of the beam at the shear span $x$

$\phi_{\text{max}}(x_0)$  - maximum curvature at shear span $x_0$

$f_c$  - compressive cylinder strength of concrete

$f_y$  - yield strength of steel element

$h_c$  - distance from centroid of concrete slab to concrete-steel interface

$h_s$  - distance from centroid of steel section to concrete-steel interface

$h_{cs}$  - distance between the centroids in steel and concrete elements ($h_{cs} = h_c + h_s$)

$h_{na}$  - distance between neutral axes in steel and concrete elements due to partial interaction

$h_{na,mid}$  - distance between neutral axes due to partial interaction at mid-span of the composite beam

$K_1, K_2$  - constants in equations deriving slip strain distribution

$K_3$  - constants in equation deriving slip strain at mid-span

$L$  - length of simply supported beam

$M(x)$  - applied moment at a distance $x$ from support

$M_{\text{max}}$  - maximum moment at the centre of the beam

$M_{\text{conc}}(x)$  - moment in concrete element at distance $x$ from support

$M_{\text{steel}}(x)$  - moment in steel element at distance $x$ from support

$M_{\text{cmp,f}}$  - composite internal moment for full-shear-connection

$P$  - point load

$P_{sb}(x)$  - strength of shear connectors in shear span $x$
$P_{sh,\text{max}}$ - maximum strength of the shear connectors at mid-span of the beam ($P_{sh,\text{max}} = q_{sh} L/2$)

$P_{\text{shear}}$ - strength of the shear connection in a shear span

$P_{fl}$ - strength of the shear connection required for full interaction

$q_{sh}$ - strength of shear connectors per unit length of beam

$R$ - radius of curvature

$S_{\text{max}}$ - maximum slip

$S_u$ - maximum slip at maximum strength of shear connector

$x$ - length of shear span

$x_{oc}$ - shear span at which curvature reaches its maximum

$x_{os}$ - shear span at which slip strain reaches its maximum

$\varepsilon$ - strain distribution

$\eta$ - degree of shear connection

$\varphi$ - degree of interaction

$\sigma$ - stress distribution

$(ds/dx)_{\text{max}}$ - maximum value of the slip strain

$(ds/dx)_{\text{mid}}$ - the slip strain at the mid-span of composite beam
1. INTRODUCTION

Composite steel and concrete construction is a relatively new form of construction as compared with both the older parent forms of steel construction and reinforced concrete construction (Oehlers and Bradford, 1995). Because composite construction was developed after these older forms of construction, design and analysis techniques have often been developed from standard practices that were established specifically for steel structures or for reinforced concrete structures. It will be shown how assumptions already endemic in the design or analysis of composite members, that originated from steel or reinforced concrete design, are not always correct when applied to composite structures with mechanical shear connectors.

This report presents a partial interaction analysis of composite beams with mechanical shear connectors in which relationships are developed between the degree of interaction and the degree of shear connection, and the slip-strain, slip distance between neutral axes.

2. THEORETICAL ANALYSIS

2.1 Behaviour of a Composite Beam

2.1.1 Curvature distribution of a composite beam

Let us consider a simply supported composite beam with a uniform distribution of shear connectors as shown in Fig. 2.1. It will be assumed that all the shear connectors are fully loaded throughout the length of the beam and that the steel and concrete elements are elastic (Oehlers and Sved, 1995). It is also assumed that the connectors prevent the separation of the steel-concrete interface so that the curvatures $\phi(x)$ in both elements at position $x$ in Fig. 2.1 are the same that is

$$\frac{M_{\text{conc}}(x)}{(EI)_c} = \frac{M_{\text{steel}}(x)}{(EI)_s} = \frac{1}{R} = \phi(x) \quad (2.1)$$

where $M_{\text{conc}}(x) =$ moment in the concrete element at $x$, $M_{\text{steel}}(x) =$ moment in the steel element at $x$, $(EI)_c =$ flexural rigidity of concrete element, $(EI)_s =$ flexural rigidity of steel element, and $R =$ radius of curvature.
Let $M(x)$ be the applied moment at section A-A of the composite beam in Fig. 2.2 which is at a distance $x$ from the supports. The internal stress resultants in the composite beam between this section and the nearest support are also shown in Fig. 2.2, where $P_{sh}$ is the shear force in the shear connectors in the shear span of length $x$, $h_{cs}$ is the distance between the centroids of the concrete and steel elements. Therefore, from equilibrium at section A-A

$$M(x) = M_{steel}(x) + M_{conc}(x) + P_{sh}h_{cs}$$

(2.2)

Fig. 2.2 Internal moments at the cross-section A-A
From Eqs. (2.1) and (2.2), a relationship for the distribution of the curvature along the composite beam $\phi(x)$ when subjected to the applied moment distribution $M(x)$ can be derived as

$$
\phi(x) = \frac{M(x) - P_{sh}(x)h_{cs}}{(EI)_c + (EI)_s} \quad (2.3)
$$

or in the simpler form:

$$
\phi(x) = \frac{M(x) - P_{sh}(x)h_{cs}}{(EI)_{cmp}} \quad (2.4)
$$

where $P_{sh}(x) = \text{strength of the shear connectors in the shear span } x$, $h_{cs} = \text{distance between centroids of the concrete and steel elements at the cross-section at } x$, and $(EI)_{cmp} = \text{flexural rigidity of the composite concrete and steel elements}$ $(EI)_c = \text{(EI)}_c + (EI)_s$.

If a uniformly distributed load is applied throughout the composite beam in Fig. 2.1, then the applied moment at the cross-section A-A at distance $x$ can be expressed as

$$
M(x) = \frac{4x(L-x)M_{\text{max}}}{L^2} \quad (2.5)
$$

where $x = \text{distance from the support}$, $L = \text{length of the span of the simply supported beam}$, and $M_{\text{max}} = \text{maximum moment at the center of the beam}$. If the shear connectors are also uniformly distributed with a strength of $q_{sh}$ per unit length of beam, then the strength of the shear connectors along the shear span from the support to $x$ can be expressed for $0 \leq x \leq L/2$ as

$$
P_{sh}(x) = q_{sh} x \quad (2.6)
$$

Substituting Eqs. (2.5) and (2.6) into Eq. (2.4) gives

$$
\phi(x) = \frac{x}{(EI)_{cmp}} \left[ \frac{4(L-x)M_{\text{max}}}{L^2} - q_{sh}h_{cs} \right] \quad (2.7)
$$

It is obvious from Eq. (2.7) that the curvature is zero at the support i.e. $\phi(0) = 0$. Now let us calculate the curvature at the centre cross-section of the beam $\phi(L/2)$ i.e. at $x = L/2$. Substituting $x = L/2$ into Eq. (2.7) gives

3
\[ \phi_{L/2} = \frac{M_{\text{max}} - P_{\text{sh,max}} h_{cs}}{(EI)_{\text{cmp}}} \]  

(2.8)

or

\[ \phi_{L/2} = \frac{M_{\text{max}} - M_{\text{cmp,max}}}{(EI)_{\text{cmp}}} \]  

(2.9)

where \( P_{\text{sh,max}} \) = maximum strength of the shear connectors at the centre of the beam \( P_{\text{sh,max}} = q_{sh} L/2 \) (from Eq. (2.6)), and \( M_{\text{cmp,max}} \) = maximum composite moment due to the shear forces (\( M_{\text{cmp,max}} = P_{\text{sh,max}} h_{cs} \)). From Eq. (2.8), it can be seen that the curvature at the centre cross-section of composite beam is dependent on the maximum value of the applied moment and the maximum strength of the shear connectors at the mid-span of the beam.

Let us investigate the distribution of the curvature as expressed in Eq. (2.7) to see when it reaches maximum value. When the first derivative \( d(\phi(x))/dx = 0 \) then

\[
\frac{1}{(EI)_{\text{cmp}}} \left[ \frac{4(L-2x)M_{\text{max}}}{L^2} - q_{sh} h_{cs} \right] = 0
\]  

(2.10a)

Solving Eq. (2.10) gives

\[ x_{oc} = \frac{L}{2} \left( 1 - \frac{M_{\text{cmp,max}}}{2M_{\text{max}}} \right) \]  

(2.10b)

where \( x_{oc} \) is the distance from the support where the curvature is the greatest. This gives the position of the cross-section at which the curvature of composite beam reaches its maximum value as the second derivative of \( \phi(x) \) is always negative \( (d^2 \phi(x)/dx^2 = -8 M_{\text{max}}/(EI)_{\text{cmp}} L^2 < 0) \).

From Eq. 2.10b, it can be seen that when there is no interaction, that is \( M_{\text{cmp,max}} = 0 \), then \( x_{oc} = L/2 \) so that the position of maximum curvature occurs at mid-span as shown in Fig. 2.3(a). As the strength of the shear connectors increase, the position of the maximum curvature moves away from mid-span as shown. The position \( x_{oc} \) is a very interesting position as it defines a transition point in a composite beam. It can be seen in Eq. 2.2, that the applied moment \( M(x) \) is resisted by flexure in the elements \( M_{\text{steel}} + M_{\text{conc}} \) and the axial component of the moment \( M_{\text{cmp}} = P_{\text{sh}} h_{cs} \). If we consider the span of beam between \( x = 0 \) and \( x = x_{oc} \) in Fig. 2.3a, as we increase \( x \) the increase in
M(x) is resisted by an increase in both $M_{\text{steel}} + M_{\text{concrete}}$ and $M_{\text{cmp}}$. However between $x = x_{oc}$ and $x = L/2$, the increase in $M_{\text{cmp}}$ is greater than the increase in $M(x)$ and, hence, the flexural component $M_{\text{steel}} + M_{\text{concrete}}$ reduces causing the curvature to reduce, which is a phenomenon that is unique to composite beams with mechanical shear connectors.

Substituting $x_{oc}$ for $x$ into Eq. (2.7) gives the following expression for the maximum curvature $\phi_{\text{max}}(x_{oc})$

$$\phi_{\text{max}}(x_{oc}) = \frac{M_{\text{max}}}{(EI)_{\text{cmp}}} \left(1 - \frac{M_{\text{cmp, max}}}{2M_{\text{max}}} \right)^2$$

(2.11)

Equations 2.9, 2.10b and 2.11 can be used to plot the variations in curvature in Fig. 2.3. Full interaction being defined in this example when the slip strains at the supports at mid-span are zero (Ref. 1) but greater than zero elsewhere except at the supports.

![Curvature distribution of composite beam subject to UDL load](image-url)

Fig. 2.3 Curvature distribution of composite beam subject to UDL load
For the particular case when $M_{\text{max}} = M_{\text{comp, max}}$ in Eq. 2.10b, then the maximum curvature is then reached at a quarter of the span length of the beam ($L/4$).

2.1.2 Longitudinal slip strain distribution of a composite beam

Let us consider a composite beam with uniformly distributed applied load and with a uniform distribution of fully loaded shear connectors as shown in Fig. 2.1. The following general equation for the slip strain distribution is obtained from Oehlerl and Sved’s work (1995)

$$
\frac{ds}{dx} = K_1 M(x) - K_2 P_{sh}(x) \quad (2.12)
$$

where $ds/dx$ = longitudinal slip strain, and where $K_1$ and $K_2$ are material constants (Oehlerl & Sved, 1995) in which $(K_1 = h_{cs} / (EI)_{\text{comp}}$, $K_2 = K_1 h_{cs} + (AE)_{\text{comp}}$ and where $(AE)_{\text{comp}} = \frac{1}{(AE)_{\text{c}}} + \frac{1}{(AE)_{\text{s}}}$.

Equation (2.12) can be rewritten in the following more specific form by using the relationship for the moment distribution due to a uniformly distributed load (Eq. 2.5) and the relationship for the strength of shear connectors along the shear span $x$ (Eq. 2.6).

$$
\frac{ds}{dx} = \frac{4M_{\text{max}}K_1x(L-x)}{L^2} - q_{sh}K_2x \quad (2.13)
$$

From Eq. 2.13, the slip strain reaches its maximum value when $d^2s/dx^2 = 0$, that is

$$
\frac{d^2s}{dx^2} = \frac{4M_{\text{max}}K_1(L-2x)}{L^2} - q_{sh}K_2 = 0 \quad (2.14)
$$

Solving Eq. (2.14) for $x$ and note that $P_{\text{sh, max}} = q_{sh} \cdot L/2$ gives

$$
x_{\text{sh}} = \frac{L}{2} \left( 1 - \frac{K_2 P_{\text{sh, max}}}{2K_1M_{\text{max}}} \right) \quad (2.15)
$$

where $x_{\text{cs}}$ = the distance from the support at which the slip strain reaches it maximum value.
It can be seen from Eq. (2.15) that the slip strain reaches its maximum at the mid-span (ie \( x_o = L/2 \)) only when \( P_{sh,\text{max}} = 0 \), that is when there is zero-shear connection between the steel and concrete elements (that is the degree of shear connection is zero, \( \eta = 0 \)). When \( \eta > 0 \), then the slip strain has its maximum value somewhere between the support and mid-span (\( x_o < L/2 \)). The following maximum value of the slip strain, \((ds/dx)_{\text{max}}\) can be obtained by substituting Eq (2.15) into Eq. (2.13) and simplifying.

\[
\left( \frac{ds}{dx} \right)_{\text{max}} = K_1 M_{\text{max}} \left( 1 - \frac{K_2 P_{sh,\text{max}}}{2 K_1 M_{\text{max}}} \right)^2 \tag{2.16}
\]

Now let us take a close look at the value of the slip strain at the mid-span, \((ds/dx)_{\text{mid}}\). Substituting \( x = L/2 \) to Eq. (2.12) gives

\[
\left( \frac{ds}{dx} \right)_{\text{mid}} = K_1 M_{\text{max}} - K_2 P_{sh,\text{max}} \tag{2.17}
\]

or

\[
\left( \frac{ds}{dx} \right)_{\text{mid}} = K_1 M_{\text{max}} - \frac{K_2 M_{\text{emp, max}}}{h_{cs}} \tag{2.18}
\]

It can be from Eq. (2.18) that the sign of the slip strain at the mid-span of composite beam depends on the relation between maximum applied moment, maximum internal composite moment, and the geometrical and the material properties of the beam.

Eq. 2.18 can be rewritten in the following simpler form.

\[
\left( \frac{ds}{dx} \right)_{\text{mid}} = K_1 (M_{\text{max}} - K_3 M_{\text{emp, max}}) \tag{2.19}
\]

where \( K_3 \) is a material constant and is given by \( K_3 = K_2 / K_1 h_{cs} \). From Eq. (2.19), it can be seen that the sign of the slip strain at the mid-span is positive when \( M_{\text{max}} / M_{\text{emp, max}} > K_3 \), and negative when \( M_{\text{max}} / M_{\text{emp, max}} < K_3 \). The transition between positive and negative slip strain at mid-span occurs when

\[
M_{\text{max}} / M_{\text{emp, max}} = K_3 \tag{2.20}
\]

This condition was referred to by Oehlers and Sved (1995) as full interaction, ie, \((ds/dx)_{\text{mid}} = 0\) at only the mid-span and supports and hence can be derived directly from Eq. 2.20.
From this analysis, the slip strain distribution along the length of composite beam is shown in Fig. 2.4(a).

**Fig. 2.4** Slip strain distribution of composite beam along its shear span subject to UDL load

2.1.3 *Longitudinal slip distribution of a composite beam*

Let us again consider a composite beam subjected to UDL with uniform distribution of fully loaded shear connectors as shown in Fig. 2.1. The longitudinal slip distribution of this beam can be determined by integrating its
slip strain distribution in Eq. (2.13) from shear span \(x\) to the midspan where the slip is zero as

\[
s(x) = \frac{4K_1M_{\text{max}}x^3}{3L^2} - \left(2K_1M_{\text{max}} - K_2P_{sh,\text{max}}\right)\frac{x^2}{L} + \left(\frac{K_1M_{\text{max}}}{3} - \frac{K_2P_{sh,\text{max}}}{4}\right)L \quad (2.21)
\]

when \(x = L/2\) Eq. (2.21) gives \(s(L/2) = 0\) as expected for the longitudinal slip at midspan of the beam. When \(x = 0\), it gives the maximum longitudinal slip at the beam support \(s_{\text{max}}\) which is consistent with previous work (Oehlers and Sved, 1995) as

\[
s_{\text{max}} = \left(\frac{K_1M_{\text{max}}}{3} - \frac{K_2P_{sh,\text{max}}}{4}\right)L \quad (2.22)
\]

The slip distribution given by Eq. (2.21) can be plotted using standard procedure for plotting a cubic polynomial as shown in Fig. 2.4(b). This maximum slip becomes \(s_{\text{max, zero}} = M_{\text{max}}K_1L/3\) for zero-interaction \((P_{sh,\text{max}} = 0)\) and \(s_{\text{max, fi}} = M_{\text{max}}K_1L/12\) for full-interaction at midspan \((K_1M_{\text{max}} = K_2P_{sh,\text{max}})\). This means that the maximum longitudinal slip at the support of a composite beam due to zero-interaction is four times of that value of the maximum slip when there is full-interaction at midspan of the beam.

It is also worth noting that the shape of the slip distribution depends on the degree of interaction as shown in Fig. 2.4(b). The transition point of the slip curve for each case (zero-interaction, partial interaction and full interaction) corresponds to the position of maximum slip strain of the \((ds/dx)\) curves respectively (Fig. 2.4(a)).

2.2 Distance between neutral axes due to partial interaction

2.2.1 General formulation

A general equation for the distance between the neutral axes in the steel and concrete elements, \(h_{na}\), due to partial interaction, can be expressed in terms of slip strain and curvature as follows:

\[
h_{na} = \frac{1}{\phi(x)} \left(\frac{ds}{dx}\right) \quad (2.23)
\]
In terms of design, the value of $h_{na}$ at the cross-section at the mid-span of a beam with full-shear-connection is of more interest and will be derived for a uniformly distributed load subsequently.

From the rigid plastic standard design procedure for shear connectors, for composite beams to reach full-shear-connection at mid-span (that is $\eta = 1$) it is necessary that

$$P_{sh,max} = P_{weak} = \min(P_{steel}, P_{concrete})$$  \hspace{1cm} (2.24)

where $P_{steel} = \text{axial strength of the steel element} = A_s f_y$, $P_{concrete} = \text{axial strength of the concrete element} = 0.85f_c A_c$, and $P_{weak} = \text{weaker of the element axial strengths}$.

Substituting Eqs. (2.9) and (2.20) into Eq. (2.23) gives

$$h_{na,mid} = \frac{K_1(EI)_{c,mp}(M_{\max} - K_3 M_{c,mp,\max})}{M_{\max} - M_{c,mp,\max}}$$  \hspace{1cm} (2.25)

Noting that $h_{cs} = K_1(EI)_{c,mp}$, then Eq. (2.25) can be rewritten in the non-dimensional form as

$$\frac{h_{na,mid}}{h_{cs}} = \frac{M_{\max} - K_3}{M_{c,mp,\max} - 1}$$  \hspace{1cm} (2.26)

where $h_{na,mid} = \text{value of } h_{na} \text{ at the cross-section at the mid-span of the beam}$.

It can be seen in Eq. 2.26 that the normalised neutral axis separation ($h_{na,mid} / h_{cs}$) can be expressed as a hyperbolic function of the normalised maximum applied moment ($M_{\max} / M_{c,mp,\max}$) as shown in Fig. 2.5 where $M_{c,mp,\max} = f(\eta)$ and, hence, Eq. 2.26 depends on the degree of shear connection. It is worth noting here that for full-shear-connection, when $M_{c,mp,\max} = P_{sh,max} h_{cs} = P_{weak} h_{cs} = \text{constant}$, then it is deduced from Eq. (2.26) that the distance between neutral axes at mid-span depends only on the maximum applied moment.

It is also worth noting that the variation in Fig. 2.5 at small values of $M_{\max}$ is illusionary as it is assumed that at small values of $M_{\max}$ the shear connectors are fully loaded which does occur in practice as the slips would not be sufficient to fully load the connectors. However, the implications are
worth considering. Let us, therefore, consider a beam which has very small ductile connectors so that they are fully loaded immediately a load is applied to the beam but are ductile enough so that they do not fracture due to excessive slip. At small values of $M_{\text{max}}$ such as at point A in Fig. 2.5 $h_{nd}/h_{cs} > 1$. This is because $M_{\text{con}} > M_{\text{max}}$ which can only occur if the curvature in both the concrete element and the steel element reverse in direction, so that the strains induced by flexure in the top fibres of each element are compressive and not tensile as would be expected.

![Diagram](image)

Fig. 2.5 Distance between neutral axes at mid-span due to partial interaction for full-shear-connection.

2.2.2 Relation between $h_{nt,mid}$ and degree of interaction $\varphi$

The degree of interaction is defined as the ratio of shear force exerted by the shear connectors in a shear span with partial interaction to the shear force required for full interaction at mid-span, that is zero slip-strain at mid-span (Oehlers and Bradford, 1995). Subsequently the relation for the degree of interaction can be written as:
\[ \varphi = \frac{P_{\text{shear}}}{P_{\text{fi}}} \]  

(2.27)

where \( P_{\text{shear}} \) = strength of the shear connection in a shear span, and \( P_{\text{fi}} \) = strength of the shear connection required for full interaction at mid-span.

Let us call \((P_{\text{sh,max}})_{\text{fi}}\) the strength of the shear connection at mid-span that is required for full interaction. Using Eq. (2.18) and noting that the slip strain due to full interaction is zero, the following relationship for \((P_{\text{sh,max}})_{\text{fi}}\) can be established as

\[ (P_{\text{sh,max}})_{\text{fi}} = \frac{K_1 M_{\text{max}}}{K_2} \]  

(2.28)

Using Eqs. (2.27) and (2.28), the relationship for the strength of the shear connection at the mid-span required for partial interaction is obtained as:

\[ (P_{\text{sh,max}})_{\text{pi}} = \frac{\varphi K_1 M_{\text{max}}}{K_2} \]  

(2.29)

Substituting Eq. (2.29) into Eq. (2.26) and subsequently simplifying gives

\[ \frac{h_{\text{na,mid}}}{h_{\text{cs}}} = \frac{K_2}{K_1 h_{\text{cs}} \varphi} - \frac{K_3}{K_2} \frac{1}{1 - \varphi} \]  

(2.30)

and noting further that \( K_3 = K_2 / K_1 h_{\text{cs}} \) then Eq.(2.30) can be rewritten as

\[ \frac{h_{\text{na,mid}}}{h_{\text{cs}}} = \frac{K_3 (1 - \varphi)}{K_3 - \varphi} \]  

(2.31)

It can be concluded from Eq. (2.31) that the distance between neutral axes at mid-span due to partial interaction depends on the degree of interaction and the material properties of the composite beam. The values of \((h_{\text{na,mid}} / h_{\text{cs}})\) are plotted as a hyperbolic function of the degree of interaction (\(\varphi\)) in Fig. 2.6.
2.2.3 Relationship between $h_{na,mid}$ and degree of shear connection $\eta$

According to the definition of degree of shear connection (Oehlers & Bradford, 1995), the degree of shear connection at mid-span can be expressed as

$$\eta = \frac{P_{sh,max}}{P_{weak}}$$

(2.32)

Based on Eq. (2.32), the relationship for $M_{cmp,max}$ can be rewritten as:

$$M_{cmp,max} = \eta \cdot P_{weak} \cdot h_{cs} = \eta \cdot M_{cmp,full}$$

(2.33)

where $M_{cmp,full}$ = composite internal moment for full-shear-connection. Substituting Eq. (2.33) to Eq. (2.26) and simplifying gives

$$\frac{h_{nc, mid}}{h_{cs}} = \frac{\frac{M_{max}}{M_{cmp,full}} - \eta K_3}{\frac{M_{max}}{M_{cmp,full}} - \eta}$$

(2.34)
Equation (2.34) is plotted in Fig. 2.7 and shows how the distance between the neutral axes at mid-span due to partial interaction varies with the degree of partial-shear-connection.

![Diagram showing relationship between neutral axes distance and degree of shear connection.](image)

Fig. 2.7. Normalised distance between neutral axes at the mid-span due to partial interaction as a function of the degree of shear connection.

Equation (2.32) can be rewritten for the condition of full interaction as

\[
\eta_{fi} = \frac{(P_{sh,\text{max}})_{fi}}{P_{\text{weak}}} \tag{2.35}
\]

where \(\eta_{fi}\) = degree of shear connection when there is full interaction, and \((P_{sh,\text{max}})_{fi}\) = strength of shear connectors required for full interaction. Substituting Eq. (2.28) into Eq. (2.35) gives

\[
\eta_{fi} = \frac{K_1 M_{\text{max}}}{K_2 P_{\text{weak}}} \tag{2.36}
\]

Furthermore, substituting Eq. (2.36) into Eq. (2.34) and subsequently simplifying gives the following better expression for the normalised distance between neutral axes due to partial interaction.
\[
\frac{h_{na,mid}}{h_{ex}} = \frac{K_3(\eta - \eta_{fi})}{\eta - K_3 \eta_{fi}}
\] (2.37)

As a result, a better graph illustrating the relationship between \(h_{na,mid} / h_{es}\) and \(\eta\) is shown in Fig. 2.8.

![Graph showing the relationship between \(h_{na,mid} / h_{es}\) and \(\eta\)](image)

Fig. 2.8. Normalised distance between neutral axes as a function of the degree of shear connection.

2.3 Relation between degrees of interaction \(\varphi\) and shear connection \(\eta\)

The following expression for the degree of interaction when there is full-shear-connection, \((\varphi)_{fsc}\), can be established using Eqs. (2.27) and (2.28).

\[
\varphi_{fsc} = \frac{(P_{sh,max})_{fsc}}{(P_{sh,max})_{fi}} = \frac{P_{weak}}{(P_{sh,max})_{fi}}
\] (2.38)

where \((P_{sh,max})_{fsc}\) = strength of shear connectors required for full-shear-connection. Substituting \((P_{sh,max})_{fi}\) from Eq. (2.28) into Eq. (2.38) gives a function for \((\varphi)_{fsc}\) and dividing this by \(\varphi\) in Eq. (2.29) gives
\[ \frac{\varphi}{\varphi_{fsc}} = \frac{(P_{sh,\max})_{pl}}{P_{weak}} = \frac{P_{sh,\max}}{P_{weak}} = \eta \quad (2.39) \]

Also it can be deduced from Eqs. (2.35) and (2.38) that

\[ \varphi_{fsc} = \frac{1}{\eta_f} \quad (2.40) \]

This means that the degree of interaction when there is full-shear-connection \((\varphi)_{fsc}\) is the inverse of the degree of shear connection when there is full interaction \(\eta_f\).

Finally, a more general relationship between the degrees of interaction and shear connection can be derived from Eqs. (2.39) and (2.40) as

\[ \varphi_{fsc} = \frac{\varphi}{\eta_f} = \frac{1}{\eta_f} \quad (2.41) \]

Let us consider an example of an application of Eq. 2.41. In standard rigid plastic analyses it is assumed that the neutral axis separation \(h_{na}\) is zero or does not affect the ultimate strength. However, Eq. 2.41 can be used to estimate \(h_{na}\) and, hence, and reduction in strength. For example, \(\eta_f\) can be derived from the applied load and the properties of the beam from Eq. 2.36 and hence \((\varphi)_{fsc}\) can be obtained from Eq. (2.41) which can be substituted into Eq. (2.31) to derive \(h_{na}\) which can then be used to determine whether the rigid plastic strength is reduced.

2.4 Expression for maximum longitudinal slip in terms of degrees of interaction \(\varphi\)

Substituting Eq. (2.29) into Eq. (2.22) and simplifying we get

\[ s_{\max}(\varphi) = K_1 M_{\max} L \left( \frac{1}{3} - \frac{\varphi}{4} \right) \quad (2.42) \]

It can be seen from Eq. (2.42) that for zero-interaction \(\varphi = 0\) the maximum longitudinal slip at the beam support \(s_{\max,\text{zero}} = K_1 M_{\max} L/3\). Similarly, for full interaction \(\varphi = 1\), Eq. (2.42) gives \(s_{\max,\bar{f}} = K_1 M_{\max} L/12\) which
is one fourth of \( s_{\text{max,zero}} \). This conclusion is consistent with that obtained in the section 2.1.3.

It is worth noting from Eq. (2.42) that the maximum longitudinal slip at the beam support depends linearly on the degree of interaction \( \varphi \).

3. SUMMARY

This report presents fundamental partial interaction analysis of composite beams with mechanical shear connectors in which relationships are developed between the degree of interaction and the degree of shear connection, and the slip-strain, slip and distance between neutral axes. This analysis can be used to estimate the reduction in strength due to the effect of partial interaction.

4. REFERENCES
