THE INFLUENCE OF FRICTION ON THE FATIGUE LIFE OF
STUD SHEAR CONNECTORS IN COMPOSITE BRIDGE BEAMS

by

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THE INFLUENCE OF FRICTION ON THE FATIGUE LIFE OF STUD SHEAR CONNECTORS IN COMPOSITE BRIDGE BEAMS

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ABSTRACT: Composite steel and concrete bridge beams are commonly used throughout the world. The steel and concrete components are typically mechanically bonded together with stud shear connectors. The nature of the loading applied to the composite bridge beams is such that there is a net normal compressive force across the steel-concrete interface, which results in a frictional force acting along the interface. The distribution of stud shear connectors, based on fatigue requirements, is currently designed without taking into account the effects of friction.

The friction mechanism acting along the interface is investigated with the use of a finite element program. The results of the finite element analyses are compared to the results obtained from a previously proposed simple hand procedure (Oehlers’ method) in an attempt to validate the procedure.
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1. INTRODUCTION

Composite bridges consist of a concrete deck supported by a series of longitudinally running steel sections. In order to improve the capacity and behaviour of the section, the steel and concrete components are commonly mechanically bonded using stud shear connectors. A cross-section of a typical composite bridge beam is shown in Fig. 1.1.

![Diagram of composite bridge beam](image)

Figure 1.1: Typical cross-section.

Although there are normal forces acting across the steel-concrete interface (axial forces), the connectors must primarily resist the longitudinal shear force per unit length, or shear flow force \( q \), acting along the interface. The mechanism by which the stud shear connectors resist the shear flow force is by dowel action and the distribution of the stud shear connectors is designed to meet the following two major criteria: the connectors must be able to resist the maximum design overload expected over the design life of the structure; and the connectors must be able to resist the fatigue loads induced by the frequent traversal of vehicles throughout the life of the bridge.

1.1 Fatigue design

The load factor and force factor parameters used in the assessment of the fatigue damage are defined in the following sections. This is followed by the presentation of the fatigue damage equation that is used to determine the distribution of shear connectors which takes into account the reduction in strength of the shear connectors upon application of cyclic loading.
This is called the crack propagation approach (Oehler and Bradford, 1995).

1.1.1 Load factor

The types of vehicles that traverse a bridge throughout its life are numerous. It is necessary to calculate the magnitude and frequency of the range of cyclic forces applied to the shear connectors in order to determine the damage caused by the traversal of each vehicle. It has been shown that the fatigue damage of shear connectors is dependent on the range of forces applied to them (Oehler and Bradford, 1995); the range of force is defined as the difference between the maximum and minimum force applied. Fatigue damage is a term used to quantify how much of the fatigue life has been used up.

It is not practical, if not impossible, to determine the stress range caused by each vehicle expected to traverse a bridge. Therefore, a standard fatigue vehicle (SFV) is used in practice. The variation in the fatigue vehicle weights is represented as a proportion of the weight of the SFV. A typical SFV is shown in Fig. 1.2.

![Standard Fatigue Vehicle](image)

Figure 1.2: Standard Fatigue Vehicle.

The magnitudes of the various fatigue vehicles are represented as a proportion of the total weight of the SFV as shown in column 2 of Table 1.1. The probability of occurrence of each fatigue vehicle is given in column 3, where the summation of column 3 must be equal to unity. The summation of column 4 is defined as the load factor $L_f$, where $m$ is the exponent of the fatigue endurance equation, which is dealt with in Section 1.1.3. Table 1.1 is called the Load Spectrum.

1.1.2 Force factor

Stresses are found using linear elastic analysis, since serviceability loads are used when dealing with fatigue. In order to simplify the design
procedure, the current practice is to assume that there is no relative displacement, or slip, between the steel and concrete components. This is known as full interaction, and assumes that the connectors are infinitely stiff so that the strain profile, at any point along the span, is linear and continuous through the depth as shown in Fig. 1.3.

<table>
<thead>
<tr>
<th>Fatigue Vehicle</th>
<th>Weight (W)</th>
<th>Probability (B)</th>
<th>BW(^m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>1</td>
<td>(W_1)</td>
<td>(B_1)</td>
<td>(B_1W_1^m)</td>
</tr>
<tr>
<td>2</td>
<td>(W_2)</td>
<td>(B_2)</td>
<td>(B_2W_2^m)</td>
</tr>
<tr>
<td>(n)</td>
<td>(W_n)</td>
<td>(B_n)</td>
<td>(B_nW_n^m)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\Sigma = 1.0)</td>
<td>(L_e = \Sigma BW^m)</td>
</tr>
</tbody>
</table>

The range of load applied to the shear connectors must be determined. This is done by obtaining the shear force influence line diagram at design points along the span. An example of an influence line is given in Fig. 1.4, for a design point at the quarter span of a simply supported beam of length \(L\). The load consists of two point loads, each of magnitude \(V\), spaced at a distance of \(L/4\). The loads are moved along the length of the beam in intervals of \(L/4\) from left to right.

![Shear force influence line diagram](image)

Figure 1.3: Full interaction strain profile.

The shear flow force influence line diagram at the steel-concrete interface for the design point shown in Fig. 1.4 is given in Fig. 1.5. The shear flow force at any point along the composite beam is calculated using the following equation,

\[
q = VA \frac{\bar{y}}{I}
\]  

(1.1)
where $V =$ the shear force, $A =$ the cross-sectional area of the concrete component, $y =$ the distance between the centroid of the concrete component and the centroid of the transformed concrete section of the composite beam, and $I =$ the moment of inertia of the transformed concrete section.

![Diagram of shear force influence line diagram](image)

Figure 1.4: Shear force influence line diagram relative to the position of the front axle.

The distribution of force in the influence line diagram must be converted to a set of equivalent cyclic forces that produce the same fatigue damage. One method of doing this is known as the reservoir method of cyclic counting (Oehlerls and Bradford, 1995). This method assumes that the influence line diagram is the cross-section of a reservoir that is to be emptied from the lowest point. The cross-section of the reservoir is obtained by drawing two shear flow influence line diagrams adjacent to each other as shown in Fig. 1.5. The distance from the top of the reservoir to the lowest point is one equivalent cyclic range, denoted by $R_1$ in Fig. 1.5. Any other remaining pockets must be drained from their lowest points until there are no more pockets remaining. Each of the remaining pockets that are drained represents an additional equivalent cyclic range. For the
example used in Fig. 1.5, there is only one more equivalent cyclic range, denoted by $R_2$.

![Shear flow force influence line diagram](image)

**Figure 1.5: Reservoir method.**

As the fatigue damage of shear connectors is dependent on the total range, both the positive and negative parts of the influence line diagram must be taken into account together.

The equivalent cyclic forces are recorded in tabular form, known as a force spectrum, shown in Table 1.2. The frequency $f_n$ in column 3 is the number of times the range $R_n$ in column 2 appears in the influence line diagram for the design point. The summation of column 4 is defined as the force factor $F_f$ which is used in the fatigue damage equation.

<table>
<thead>
<tr>
<th>Range Number (1)</th>
<th>Range (R) (2)</th>
<th>frequency (f) (3)</th>
<th>$fR^m$ (4)</th>
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<tr>
<td>1</td>
<td>$R_1$</td>
<td>$f_1$</td>
<td>$f_1R_1^m$</td>
</tr>
<tr>
<td>2</td>
<td>$R_2$</td>
<td>$f_2$</td>
<td>$f_2R_2^m$</td>
</tr>
<tr>
<td>$n$</td>
<td>$R_n$</td>
<td>$f_n$</td>
<td>$f_nR_n^m$</td>
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$F_f = \sum fR^m$
1.1.3 Fatigue damage equation

The fatigue damage equation is derived from the following generic form of the endurance equation (Oehlers and Bradford, 1995),

\[ E = C \left( \frac{R}{P_{st}} \right)^m \]  \hspace{1cm} (1.2)

where \( E \) = the endurance, or number of cycles required to cause failure at range R, \( C \) and \( m \) = fatigue material properties that are determined experimentally, and \( P_{st} \) = the static strength of the connectors. For the case of stud shear connectors, it has been found that \( C \) is equal to \( 10^{3.12} \), and \( m \) is equal to 5.1 (Oehlers and Bradford, 1995).

The following generic form of the accumulated damage law is also required in the development of the fatigue damage equation (Oehlers and Bradford, 1995),

\[ \sum \frac{N}{E} = I - \frac{P_{res}}{P_{st}} \] \hspace{1cm} (1.3)

where \( P_{res} \) = the residual, or remaining strength of the shear connectors after the application of cyclic loads. The left hand side of Eq. 1.3 is a measure of the fatigue life that is used up after \( N \) applications of load, and the right hand side takes into account the reduction in strength of the shear connectors.

Combining Equations 1.2 and 1.3 results in the fatigue damage equation for stud shear connectors given by Eq. 1.4 (Oehlers and Bradford, 1995). This form of the fatigue damage equation is such that it can be used to design the stud shear connectors in new composite bridge beams.

\[ Q^{-5.1} = \left( I - \frac{q_o}{Q} \right) 10^{3.12} \]

\[ \frac{\sum_{i=1}^{n} (TF_f L_f)_i}{\sum_{i=1}^{n} (TF_f L_f)_i} \] \hspace{1cm} (1.4)

where \( n \) = the number of standard fatigue vehicles, \( T_i \) = the number of traversals of SFV, \( q_o \) = the maximum shear flow force caused by the maximum overload, and \( Q \) = the shear flow strength that the shear connectors must have at the start of the design life. The denominator of
the right hand side of Eq. 1.4 is called the fatigue damage term. An iterative procedure is required to solve for \( Q \), and the equation ensures that the shear flow strength is always greater than the maximum shear flow force throughout the life of the composite bridge beam.

When assessing the performance of existing composite bridges, some of the variables in Eq. 1.4 are altered.

### 1.2 Interfacial friction

Due to the orientation of the applied loads on composite bridge beams and the weight of the concrete deck, the net normal force acting across the steel-concrete interface must be compressive. Situations may arise, however, where uplift can take place locally along the steel-concrete interface of composite bridge beams (Johnson, 1975). The resulting normal tensile forces, however, are small compared to the shear forces that must be resisted. Therefore, it is often ignored in design, especially when connectors with uplift resistance are used, as is the case with stud shear connectors. The effect of normal tensile forces can be accounted for, however, by reducing \( P_s \) (Oehlers and Bradford, 1995). Similarly, the effect of compressive normal forces can be accounted for by increasing \( P_s \).

The normal compressive force results in a frictional force acting along the steel-concrete interface. The frictional resistance must first be overcome before the shear connectors slip and begin to carry load. As a result, the shear flow force that must be resisted by dowel action of the shear connectors is reduced by an amount equal to the frictional flow, given by the following equation:

\[
q_{dwl} = q_{total} - q_{fric}
\]  

(1.5)

where \( q_{fric} \) = the frictional resistance per unit length, and \( q_{dwl} \) = the shear flow force resisted by the shear connectors. This is represented schematically in Fig. 1.6, for a shear span of length \( L \).

The reduction in the shear flow force resisted by the connectors results in a reduction of the cyclic range of load. This in turn reduces the magnitude of the force factor \( F_c \). The implication of this is that the design life of stud shear connectors may be greater than that currently assumed.
1.2.1 Proposed design procedure

A simple hand assessment procedure has already been developed (Oehlers and Bradford, 1995) that allows for the reduction in the shear flow force resisted by the shear connectors due to the beneficial effect of friction. This method will be referred to as Oehlers’ method, and a description of the procedure follows.

The problem that was encountered in the development of the model was how to determine the magnitude of the frictional force along the length of the steel-concrete interface. The assumption that was made to overcome this problem was that the frictional resistance is uniform throughout the shear span. This implies that the normal force across the steel-concrete interface is uniform along the shear span as well. Therefore, the frictional flow along the interface of a shear span is given by the following equation,

\[ q_{fric i} = \frac{\mu V_i}{L_i} \]  \hspace{1cm} (1.6)

where \( \mu \) = the coefficient of friction between the steel and concrete, \( V_i \) = the shear force acting on shear span \( i \), and \( L_i \) = the length of shear span \( i \). Figure 1.7 illustrates this assumption for the case of a single point load acting on a simply supported composite bridge beam. This assumption also implies that all of the frictional resistance in a shear span must be overcome before the shear connectors begin to resist any of the shear flow force.
Figure 1.7: Distribution of frictional forces.

By substituting Eqs 1.1 and 1.6 into Eq. 1.5, the following mathematical model is developed for shear span \( i \), which calculates the shear flow force resisted by the connectors taking into account the effects of friction.

\[
q_{dwi, i} = V_i \left( \frac{A_y}{l} - \frac{\mu}{L_i} \right)
\]  
(1.7)

1.2.2 Coefficient of friction

In order to take into account the effects of friction, the coefficient of friction along the steel-concrete interface must be known. The variation of the coefficient of friction between steel and concrete subjected to cyclic loading was determined experimentally (Singleton, 1985). The tests consisted of applying up to 4 million cyclic displacements to a steel plate located between two concrete blocks. The normal compressive force was applied through large springs. The experimental set up is shown graphically in Fig. 1.8.

It was observed that the coefficient of friction fluctuated during the tests, consisting of a series of peaks and troughs. An initial increase was due to the wearing away of the weaker surface of the concrete block exposing the coarse, harder concrete below the surface. Additional cycles gradually wears away and polishes the surface causing a reduction in the coefficient of friction, until the coarse aggregate is once again exposed resulting in another increase in the coefficient of friction.
It was found that the coefficient of friction along the steel-concrete interface varied between 0.70 and 0.95 when subjected to cyclic loading.

1.3 Validity of design procedure

Before Eq. 1.7 can be adopted in practice, the assumptions and results must be verified. This has been done by developing a finite element program suitable for modelling the behaviour of composite bridge beams. The details of the program are given in Section 2 of this report. The mechanism by which friction influences the shear flow force resisted by the shear connectors, as observed from the finite element analyses, is presented in Section 3. Section 4 compares the results of Oehlers’ method with those of the finite element analyses. The conclusions are summarised in Section 5.

1.4 Partial interaction

When the true stiffness of the connectors is used in an analysis, slip along the interface occurs. This approach is known as partial interaction, and the implication is that the strain profile through the depth of the composite beam is no longer continuous. There is a discontinuity at the steel-
concrete interface due to the slip, which is defined as the slip strain (ds/dx) as shown in Fig. 1.9.

![Diagram showing partial interaction strain profile](image)

**Figure 1.9:** Partial interaction strain profile.

The frictional resistance prevents slip until the total longitudinal shear force is large enough to overcome it at which time slip occurs and the connectors begin to resist a portion of the longitudinal shear force.

It has been found, however, that a partial interaction analysis results in a significant reduction in the shear flow force resisted by the shear connectors. This is an important point and will be dealt with separately in future work. Nevertheless, the results using Oehler’s method are compared to those from the computer simulations.

2. **COMPUTER MODEL**

The steel and concrete components of the composite bridge are modelled using standard 4-noded isoparametric elements (Cheung and Yeo, 1979), with two translational degrees of freedom per node. The steel and concrete stiffnesses, $E_s$ and $E_c$ respectively, are defined in the input data and remain constant throughout the analysis.

The shear connectors at the steel-concrete interface are modelled by two orthogonal spring elements, as shown in Fig. 2.1.

The vertical spring represents the axial stiffness and the horizontal spring models the shear stiffness of the connectors. The set of springs at each nodal point along the steel-concrete interface represents the group of connectors distributed within that node’s tributary length.

The axial spring stiffness is made large relative to the shear stiffness, and has a minimal effect on the flexural behaviour of the beam. The shear stiffness, when taking into account the effects of friction, is determined using an iterative secant stiffness approach described in the following section.
2.1 Shear stiffness

The shear stiffness of a spring, representing a group of connectors, is based upon the type of analysis being performed and the axial force acting on the spring. The two types of analysis procedures possible are the full interaction analysis (FIA) and the partial interaction analysis (PIA), as described in Sections 1.1.2 and 1.4 respectively.

In order to prevent slip from occurring in a FIA, the stiffness of the horizontal springs is made sufficiently large so that the slip is minimised. With a PIA, however, the realistic shear stiffness based on the connector distribution is used. The initial stiffness of a stud shear connector is determined using the equations given below (Oehler and Bradford, 1995),

\[
P_{st} = 4.3A_{sh}f_u^{0.65}f_c^{0.35}\left(\frac{E_c}{E_s}\right)^{0.40}
\]

\[
K_{si} = \frac{P_{si}}{\frac{d_{sh}(0.16 - 0.0017f_c)}{}}
\]

where \(P_{st}\) = the shear strength of a stud shear connector in a composite beam, \(K_{si}\) = the initial shear stiffness of the stud shear connector, \(d_{sh}\) = the shank diameter, \(A_{sh}\) = the cross-sectional area of the shank, \(f_c\) = the concrete compressive strength, and \(f_u\) = the ultimate tensile strength of the stud shear connector.

If the normal force \(F_{norm}\) acting on a group of connectors is tensile, there is no frictional resistance \(F_{fric}\) and the original shear stiffness is retained. If, however, \(F_{norm}\) is compressive, then \(F_{fric}\) is calculated by the following equation,
\[ F_{\text{fric}} = \mu F_{\text{norm}} \tag{2.3} \]

If \( F_{\text{fric}} \) is greater than the total longitudinal shear force \( F_{\text{total}} \) acting on the group of connectors, slip is prevented and the spring shear stiffness used in the PIA is increased to the same order of magnitude used in a FIA where interfacial slip is prevented. When \( F_{\text{fric}} \) is less than \( F_{\text{total}} \), the connectors are required to resist the shear force \( F_{\text{dwl}} \) that is in excess of \( F_{\text{fric}} \), given by the following equation,

\[ F_{\text{dwl}} = F_{\text{total}} - F_{\text{fric}} \tag{2.4} \]

The shear secant stiffness of each spring, representing a group of connectors, is found by way of the iterative procedure that follows. The frictional force distribution is not known in the first iteration, so the initial spring stiffnesses \( K_{si} \) are used to perform the analysis (line a, Fig. 2.2). The results of the first iteration provide the slip \( \delta \) and normal force at each spring location from which the frictional force at each spring location can be determined using Eq. 2.3. A better estimate of the spring secant shear stiffness \( K_{\text{sec,new}} \) is then made at each spring location by defining a new load path for the group of connectors at a node (line b).

Figure 2.2: Shear secant stiffness at a nodal point.
The connectors do not begin to resist any load until the frictional resistance is overcome (point A, Fig. 2.2) at which time slip occurs and the connectors begin to carry load. It is assumed that the stiffness of the connectors has not changed after the application of load, so the initial shear stiffness of the connectors is retained. Using the new load path, $K_{sec_{new}}$ is found based on the new total longitudinal shear force $F_{total}$ required to produce the same slip $\delta$ (point B). The new spring secant shear stiffness (line c) is defined as the slope of the line that passes through the origin and point B, given by the following equation,

$$K_{sec_{new}} = \frac{F_{total}}{\delta}$$  \hspace{1cm} (2.5)

The analysis is repeated for the second iteration once all of the spring secant shear stiffnesses have been updated. Iterations continue until the maximum difference in the secant shear stiffness of the springs, in successive iterations, has met the convergence limit prescribed by the user.

In order to calculate the range of cyclic forces acting on the connectors, the applied loads are moved across the composite bridge beam in a finite number of locations so that the influence line diagram at each design point may be calculated. The loads are applied to the nodal points on the top surface of the concrete elements and are moved along the composite beam by shifting each nodal load from the current node to the adjacent node.

3. THE FRICTION MECHANISM

A 20 m long simply supported composite beam is used to illustrate the effect of friction along the steel-concrete interface under varying load conditions. The cross-sectional geometry of the composite beam is shown in Fig. 3.1. A uniform distribution of connectors, equivalent to 3 rows of 19 mm stud shear connectors spaced at 120 mm, is used. In all of the following analyses, a friction coefficient of 0.8 was used.

3.1 Single Concentrated Load

Figures 3.2 to 3.7 show the results of the analyses both with and without friction for ease of comparison.
Figure 3.1: Cross-section of 20 m long composite beam.

Figure 3.2 shows the distribution of normal forces acting along the steel-concrete interface for a concentrated load of 80 kN located at the quarter span. Large compressive forces are concentrated in a narrow region surrounding the point load and the supports. Adjacent to these regions small tensile zones exist, and the zones away from the load and supports are essentially not exposed to any normal forces across the interface. The integration of the normal force distribution along the interface is equal to the total force applied to the top surface of the composite beam in order to maintain a state of equilibrium. The normal force distribution across the interface is clearly very localised, contrary to the assumption made in the development of Oehlers’ method. Figure 3.2 also shows that there is no significant effect on the normal force distribution when friction is taken into account in the analysis.

Figure 3.3 shows the shear flow forces acting along the steel-concrete interface. Line a is $q_{drel}$ when friction is ignored. Line b is $q_{total}$ and line c is $q_{drel}$ when friction is included in the analysis. The frictional resistance, therefore, is equal to the difference between lines b and c. The point where the shear flow force changes direction is labelled the transition point.

Where the normal force across the interface is compressive (Fig. 3.2), the frictional resistance is large. The connectors, however, continue to resist longitudinal shear forces in these regions of high frictional resistance as shown by line c, that is these regions of the steel-concrete interface do not lock up when subjected to large frictional resistances. Figure 3.4 verifies that no region of the beam locks up as there is a continuous distribution of slip across the span both with and without friction. The slip is equal to zero only at the transition point. As the
connectors must slip in order to resist the longitudinal shear flows, the slip is large where $q_{dw}$ is large (Fig. 3.3).

![Figure 3.2: Normal flow forces.](image)

In the left shear span (Fig. 3.3), $q_{dw}$ is reduced relatively uniformly across the span when friction is taken into account in the analysis. However, in the right shear span, $q_{dw}$ is increased over part of the shear

![Figure 3.3: Shear flow forces.](image)
span when friction is included. The very high compressive forces under the concentrated load (Fig. 3.2) resulting in the large frictional resistance in the area (Fig. 3.3) has the effect of moving the transition point towards the load. The shift of the transition point to the left increases $q_{dwl}$, and hence the slip, in the right shear span.

![Interfacial slip graph]

**Figure 3.4: Interfacial slip.**

3.1.1 *Moving concentrated load*

The range of load resisted by the shear connectors $R_{dwl}$ when subjected to a concentrated load of 80 kN moving across the 20 m long composite beam (Fig. 3.1) is given in Fig. 3.5. Line $a$ is $R_{dwl}$ without friction, and line $b$ is $R_{dwl}$ with friction.

When friction is incorporated into the analysis $R_{dwl}$ is reduced in the vicinity of the supports ($A$-$B$, Fig. 3.5), but is increased over the remainder of the beam. The distribution of $R_{dwl}$ in Fig. 3.5 is determined from the shear flow force influence line diagrams at design points along the beam.

Figure 3.6 is the shear flow force influence line diagram at the design point located 2.5 m away from the left support, and Fig. 3.7 is at a design point 5 m away. At the 2.5 m design point, within region $A$-$B$ in Fig. 3.5, the peak negative $q_{dwl}$ is reduced significantly due to friction when the load is at 5 m (Fig. 3.6). As a result, $R_{dwl}$ with friction is less than $R_{dwl}$ without friction for the design point at 2.5 m.
Figure 3.5: $R_{dwl}$ for a moving concentrated load of 80 kN.

When the load is at 3 m in Fig. 3.7, the peak positive $q_{dwl}$ at the 5 m design point is increased significantly due to the shift of the transition point (Fig. 3.3). The peak negative $q_{dwl}$ at the 5 m design point is reduced when the load is at 8 m due to the effects of friction. Since the increase in $q_{dwl}$ due to the shift of the transition point is greater than the reduction due to the effects of friction, $R_{dwl}$ with friction is greater than $R_{dwl}$ without friction at the 5 m design point.

The increase in $R_{dwl}$ over most of the beam (Fig. 3.5) has serious implications. Since the force factor $F_f$ (Section 1.1.2) is dependent on the magnitude of $R_{dwl}$ raised to the exponent $m$, even a small change in $R_{dwl}$ will result in a large change in $F_f$. When analysing the effect of increasing $F_f$ on the fatigue damage equation (Eq. 1.4), it is evident that the fatigue life of the stud shear connectors would be less than originally expected if the distribution was designed ignoring friction. A desirable outcome of the analysis in which friction is taken into account is that $R_{dwl}$ has become more uniform over the length of the beam (line $b$, Fig. 3.5). Other phenomena such as incremental set (Oehler and Bradford, 1995) may also have the effect of making the distribution of $R_{dwl}$ more uniform.

A single concentrated load traversing a composite bridge beam may not appear to be a suitable representation of a fatigue vehicle when performing fatigue type analyses. However, this condition is approached when a fatigue vehicle with a small wheel base traverses a long composite bridge. Composite bridges as long as 50 m are not uncommon (Iles, 1991). The following section describes the behaviour along the steel-
concrete interface when the SFV (Fig. 1.2) traverses the 20 m long composite beam.

![Graph showing shear force influence line diagram at 2.5 m.](image1)

*Figure 3.6: Shear flow force influence line diagram at 2.5 m.*

![Graph showing shear force influence line diagram at 5 m.](image2)

*Figure 3.7: Shear flow force influence line diagram at 5 m.*
3.2 Standard Fatigue Vehicle

The results of the analyses performed when the SFV traverses the 20 m long composite bridge beam (Fig. 3.1) are described in this section. Due to the length of the SFV, all of the axles are not always on the bridge span at one time. Figures 3.8 to 3.10 show the normal flow force, shear flow force, and interfacial slip distributions for the load condition where the leading axle is located 5 m from the left support. Since the SFV is moving from the left to the right, only the first two axles are on the composite bridge beam. Figures 3.11 to 3.13 show the normal flow force, shear flow force, and interfacial slip distributions for the load condition where the leading axle is located at the midspan. Under this load condition, all of the axles are located on the composite bridge beam. In all of the figures, the results of the analyses both with and without friction are shown for comparison.

The distribution of the normal forces across the interface, Figures 3.8 and 3.11, is similar to that observed for the single concentrated load. High compressive forces are located in the zone surrounding the concentrated loads and near the supports. As expected, taking into account friction has essentially no effect on the normal force distribution.

![Diagram of normal flow forces - SFV at 5 m.](image)

The transition point, when the SFV is at 5 m (Fig. 3.9), is shifted towards the leading axle increasing $q_{awl}$ (line c) in the span to the right of the leading axle. The increased compressive normal forces across the
interface in the left shear span, due to the additional axle load, has significantly decreased $q_{dwl}$ as a result of the frictional resistance.

![Graph showing shear flow forces](image1)

**Figure 3.9:** Shear flow forces - SFV at 5 m.

The slip along the interface when the leading axle is located at 5 m (Fig. 3.10) is also increased in the right shear span when friction is included (line b) due to the relationship between the slip and $q_{dwl}$. Figure 3.10 shows that the distribution of slip is again continuous along the span, even though the frictional forces are very large between the two axle loads.

![Graph showing interfacial slip](image2)

**Figure 3.10:** Interfacial slip - SFV at 5 m.
For the load case where the leading axle of the SFV is located at the midspan (Fig. 3.11), \( q_{dwl} \) is reduced along the entire length of the beam (line c, Fig. 3.12). The distribution of axle loads to the left of the leading axle greatly reduces \( q_{dwl} \) in those shear spans. There is no increase in \( q_{dwl} \) in the right shear span under this load case since the shift of the transition point is minimal when it is near the midspan.

![Graph showing normal flow forces - SFV at 10 m.](image)

**Figure 3.11: Normal flow forces - SFV at 10 m.**

![Graph showing shear flow forces - SFV at 10 m.](image)

**Figure 3.12: Shear flow forces - SFV at 10 m.**
As the SFV travels from the left support towards the midspan of the composite beam, the transition point also moves towards the midspan. As this happens, the distance by which the transition point is shifted to the left, when friction is taken into account, reduces. The distance between the transition points must diminish because the transition points have to coincide when the loads are symmetrically located about the centreline of the composite beam. As the SFV continues to move to the right, the transition points again diverge, now being shifted to the right when friction is accounted for.

The interfacial slip distribution (Fig. 3.13) illustrates again that segments of the interface do not lock up, especially where the frictional forces are very large between the left-most axle and the left support. The distribution also shows that the shift of the transition point is minimal, and that the slip is reduced along the entire length of the beam.

![Interfacial slip - SFV at 10 m.](image)

**Figure 3.13: Interfacial slip - SFV at 10 m.**

### 3.2.1 Moving Standard Fatigue Vehicle

The shear flow force influence line diagram for the design point located at 5 m from the left support for the traversal of the SFV is shown in Fig. 3.14. The increase in the peak positive $q_{dwl}$ is less than the decrease in the peak negative $q_{dwl}$. This occurs at all design points under this load condition and results in an overall reduction in $R_{dwl}$ when friction is accounted for.
Figure 3.14: Shear flow force influence line diagram at 5 m.

From the shear flow force influence line diagrams calculated at each design point along the beam, the distribution of $R_{dW}$ resulting from the traversal of the SFV is shown in Fig. 3.15. The range of load resisted by the connectors with friction (line $b$) is less than the range of load resisted by the connectors without friction (line $a$) along the entire length of the beam. The reduction, under this load condition, is beneficial because the fatigue life of the connectors would be longer than originally expected assuming that the distribution was designed ignoring friction.

Figure 3.15: $R_{dW}$ for a moving SFV.
4. COMPARISON OF RESULTS

The change in $R_{dwl}$ when friction is taken into account alters the fatigue life of the stud shear connectors. The aim of this section is to determine how $R_{dwl}$ with friction using Oehlers’ method compares to the results of the computer simulations. Oehlers’ method is a simple hand procedure that is based on a full interaction analysis, while the computer results are based on the more realistic behaviour of partial interaction.

4.1 20 m Composite Beam

A comparison of the results for the 80 kN concentrated load traversing the 20 m long composite beam (Fig. 3.1) is shown in Fig. 4.1. The results of Oehlers’ method are superimposed on the results of the finite element analysis that are shown in Fig. 3.5. The design points used in Oehlers’ method are located at the two end supports, at 2.5 m and 5 m from the supports, and at the midspan.

![Figure 4.1: $R_{dwl}$ - Single 80 kN load and 20 m long composite beam.](image)

The range of load resisted by the stud shear connectors in a FIA ignoring friction (line c, Fig. 4.1) is constant along the span for the traversal of a single point load on a simply supported beam. The decrease in $R_{dwl}$ when friction is accounted for using Oehlers’ method is greatest near the supports. The normal force across the interface is high when the load is near the supports and the length of the shear span is short resulting
in a high frictional resistance. The difference between line c and $R_{dwl}$ with friction using Oehlers’ method is a measure of the frictional resistance assumed at that design point.

The distribution of $R_{dwl}$ with friction using Oehlers’ method is considerably different from the distribution obtained from the finite element analyses (line b, Fig. 4.1). The primary reason for the difference is that Oehlers’ method is derived from a FIA while the computer simulations use a PIA. The results obtained using Oehlers’ method are, however, conservative along the length of the beam. The reduction in $R_{dwl}$ with friction compared to $R_{dwl}$ that is currently used in design (line c) indicates that the fatigue life of all the stud shear connectors is greater than that currently assumed in design.

Figure 4.2 compares the results of Oehlers’ method to the finite element results shown in Fig. 3.15 for the load case of the SFV traversing the 20 m long composite beam. The distribution of $R_{dwl}$ from a FIA is shown (line c) as well as $R_{dwl}$ with friction using Oehlers’ method for the design points shown in Fig. 4.2. The distributions of $R_{dwl}$ obtained from partial interaction finite element analyses both with and without friction are given by lines b and a respectively.

![Figure 4.2: $R_{dwl}$ - SFV and 20 m long composite beam.](image)

Oehlers’ method again does not predict the same distribution of $R_{dwl}$ with friction as the computer simulation, however, the predictions are still conservative over most of the span. There is some discrepancy near the supports, but this is attributed to the disturbance caused by the support conditions in the finite element analyses. The distribution of $R_{dwl}$ with
friction using Oehlers’ method is less than the distribution used in practice (line c) resulting in a design life longer than originally anticipated.

4.2 50.4 m Composite Beam

A 50.4 m long simply supported composite beam was designed to investigate the behaviour along the steel-concrete interface when the SFV given in Fig. 1.2 is moved across the beam. The cross-sectional geometry of the beam is given in Fig. 4.3. A uniform distribution of connectors, equivalent to a single row of 19 mm diameter studs spaced at 300 mm, is used.

![Figure 4.3: Cross-section of 50.4 m long composite beam.](image)

The distribution of $R_{dvl}$ with and without friction resulting from the computer simulation of the traversal of the SFV, is given by lines b and a respectively in Fig. 4.4. Over most of the span, $R_{dvl}$ with friction (line b) is reduced, except for two narrow regions located at approximately the quarter and three-quarter spans of the beam. The length of the SFV compared to the 50.4 m length of the composite beam is sufficiently small.
such that the condition of a point load traversing the beam is being approached, resulting in some regions where $R_{dwl}$ is increased. The increase in $R_{dwl}$ with friction occurs when the increase in the peak positive $q_{dwl}$, due to the shift of the transition point, is greater than the reduction in the peak negative $q_{dwl}$ due to friction.

![Graph showing $R_{dwl}$ vs X [m]](image)

**Figure 4.4: $R_{dwl}$ - SFV and 50.4 m long composite beam.**

The distribution of $R_{dwl}$ without friction given by a FIA is given by line $c$ in Fig. 4.4. The magnitude of $R_{dwl}$ with friction using Oehlers’ method is given for the design points shown in Fig. 4.4. As was the case with the 20 m long composite beam, Oehlers’ method, based on a FIA, is not capable of predicting the distribution of $R_{dwl}$ with friction accurately. The predicted values are, however, conservative over the full length of the beam, and are less than the distribution of $R_{dwl}$ without friction (line $c$) currently used in design. The reduction in $R_{dwl}$ resulting in a longer than anticipated fatigue life for the stud shear connectors.

5. CONCLUSIONS

1. With respect to the distribution of $R_{dwl}$ obtained from a FIA, the fatigue life of stud shear connectors in simply supported composite bridge beams is calculated to be longer when friction along the steel-concrete interface is taken into account.
2. The distribution of compressive forces across the interface is concentrated within a narrow region surrounding the concentrated axle loads and the end supports. Tensile forces of very small magnitude are found adjacent to the concentrated compressive forces, with the remainder of the interface being subjected to essentially no normal force.

3. The extremely high frictional forces along the interface, resulting from the concentrated normal compressive force distribution, do not prevent slip over segments of the interface. The distribution of slip remains continuous over the entire length of beam.

4. In partial interaction analyses with friction, the transition point is shifted away from the midspan of the beam towards the axle loads. This results in an increase in $q_{dwl}$ in the shear span(s) located on the midspan side of the transition point.

5. In a partial interaction analysis, the frictional forces along the interface reduce $q_{dwl}$ relatively uniformly in the shear span(s) located on the end support side of the transition point.

6. The amount by which $R_{dwl}$ is decreased, or increased, (in a PIA) is dependent on the ratio of the length of the SFV to the length of the composite bridge beam. The shorter the SFV is compared to the length of the beam, the smaller the reduction in $R_{dwl}$ is, until a point is reached where $R_{dwl}$ is increased over regions of the beam.

7. Oehlers’ method, taking into account friction by assuming that the normal compressive force is uniformly distributed across the shear span, appears to be an upper bound model of the distribution of $R_{dwl}$. The predicted $R_{dwl}$ with friction tends to be conservative when compared to the realistic distribution obtained from the computer simulations using a PIA.

6. REFERENCES


7. NOTATION

\begin{align*}
A & \quad \text{cross sectional area of the concrete component} \\
A_{sh} & \quad \text{cross sectional shank area of a stud shear connector} \\
B & \quad \text{probability of occurrence of the fatigue vehicle} \\
C & \quad \text{fatigue material property determined experimentally} \\
ds/dx & \quad \text{slip strain} \\
d_{sh} & \quad \text{shank diameter of a stud shear connector} \\
E & \quad \text{endurance (the number of cycles required to cause failure)} \\
E_c & \quad \text{the modulus of elasticity of concrete} \\
E_s & \quad \text{the modulus of elasticity of steel} \\
F_{dwl} & \quad \text{shear force resisted by the stud shear connectors} \\
F_f & \quad \text{force factor} \\
FIA & \quad \text{full interaction analysis} \\
F_{norm} & \quad \text{normal force across the steel-concrete interface} \\
F_{fric} & \quad \text{frictional force along the steel-concrete interface} \\
F_{total} & \quad \text{the total longitudinal shear force along the steel-concrete interface} \\
f & \quad \text{frequency} \\
f_c & \quad \text{concrete compressive strength} \\
f_u & \quad \text{ultimate tensile strength of a stud shear connector} \\
I & \quad \text{moment of inertia of the transformed concrete section} \\
K_{sec,new} & \quad \text{the new estimate of the shear secant stiffness for a group of connectors} \\
K_{si} & \quad \text{initial shear stiffness of a stud shear connector} \\
L & \quad \text{span length of a simply supported beam; shear span length} \\
L_f & \quad \text{load factor} \\
m & \quad \text{fatigue material property determined experimentally}
\end{align*}
N  the number of cyclic load applications
N.A. neutral axis
P  magnitude of a concentrated load
PIA partial interaction analysis
\( P_{\text{res}} \) residual strength of a stud shear connector after \( N \) applications of load
\( P_{\text{st}} \) static strength of a stud shear connector
Q  shear flow strength of stud shear connectors at the start of the design life
q  shear flow force
\( q_{\text{dwl}} \) shear flow force resisted by the stud shear connectors
\( q_{\text{frie}} \) frictional flow force
\( q_0 \) maximum shear flow force caused by the maximum design overload
\( q_{\text{total}} \) total longitudinal shear flow force
R  range of cyclic load
\( R_{\text{dwl}} \) range of cyclic load resisted by the stud shear connectors
SFV standard fatigue vehicle
T  the number of traversals of the SFV
V  magnitude of a concentrated load; shear force
W  weight of a fatigue vehicle as a proportion of the weight of the SFV
\( y \) the distance between the centroid of the concrete component and the centroid of the transformed concrete section
\( \delta \) interfacial slip
\( \varepsilon \) strain
\( \phi \) curvature
\( \mu \) coefficient of friction