MATHEMATICAL MODELS FOR FLEXURAL PEELING
OF ANGLE PLATES GLUED TO RC-BEAMS

by

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ABSTRACT: This work on gluing steel angle plates to the sides of existing reinforced concrete beams is a further development of earlier research on gluing steel plates to the soffits (Ref. 1) and to the sides of concrete beams (Ref. 2 & 3). Generic mathematical models have been developed for simulating the debonding of angle plates glued to the edges of RC-beams due to flexural peeling. These models have been validated with experimental results and a rational design approach has been proposed to prevent debonding.
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1. INTRODUCTION

Design approaches have been developed for gluing steel plates to the soffits and sides of reinforced concrete beams (Refs. 1 & 2). In this study, a mathematical model for preventing debonding of glued angle sections due to flexural forces is developed.

2. FORCES ACTING ON ANGLE PLATES GLUED TO RC-BEAM

Let us consider a composite RC-beam that is subjected to a two point load and in which the angle plates are terminated within a constant moment region as shown in Fig. 1.

![Beam Geometry](image)

**Fig. 1. Beam Geometry: (a) Beam set up; (b) Beam cross-section**

Let us consider a cross-section of the plated beam at a distance equal to the depth of the plate \( h_p \) from the end of the side plate as shown in Fig. 2(b). In this analysis, it is assumed that the behaviour of the composite beam is linear elastic as we are dealing with the debonding of real structures where the plates are terminated a long way from the position of maximum moment.

From Fig. 2(b), the strain at the centroidal axis position in the plate \( (\varepsilon_p) \) is

\[
\varepsilon_p = h_{p,cmp} \phi
\]

where \( h_{p,cmp} \) is distance between centroids of the angle plate and composite cross-section and \( \phi \) is the curvature of the composite cross-section at distance \( h_p \) from the angle plate end (Fig. 2(b)).
Hence, the axial force acting on the angle plate can be expressed as

\[ F_p = A_p (E\varepsilon)_p = (EA)_p h_{p,cnp} \phi \]  

(2)

where \( A_p \) is the cross-section area of the angle plate.

As

\[ \phi = \frac{M_{cnp}}{(EI)_{cnp}} \]  

(3)

substituting Eq. (3) into Eq. (2) gives

\[ F_p = \frac{(EA)_p h_{p,cnp} M_{cnp}}{(EI)_{cnp}} \]  

(4)

Fig. 2. Strain profile of angle plates glued to sides of RC-beam

Now let us consider the forces acting on each element as shown in the free body diagram in Fig. 3. As there are two angle plates glued to sides of the beam, the axial force in the reinforced concrete element will be twice the axial force acting on each angle-plate that is

\[ F_{RC} = 2 F_p \]  

(5)

From equilibrium of moments, we get

\[ M_{cnp} = M_{RC} + 2 M_p + 2 F_p h_{p,RC} \]  

(6)
and from the compatibility of the curvatures of the plate and concrete elements

\[ \phi = \frac{M_{RC}}{(EI)_{RC}} = \frac{M_p}{(EI)_p} = \frac{M_{RC} + 2M_p}{(EI)_{RC} + 2(EI)_p} \]  

(7)

From Eqs. (6) and (7), a new relationship for \( \phi \) can be derived as

\[ \phi = \frac{M_{cmp} - 2F_p h_{p,RC}}{(EI)_{RC} + 2(EI)_p} \]  

(8)

and substituting Eq. (2) into Eq. (8) gives

\[ \phi = \frac{M_{cmp} - 2\phi(EA)_p h_{p,RC} h_{p,cnp}}{(EI)_{RC} + 2(EI)_p} \]  

(9)

and rearranging Eq. (9) and solving for \( \phi \) gives

\[ \phi = \frac{M_{cmp}}{(EI)_{RC} + 2(EI)_p + 2(EA)_p h_{p,RC} h_{p,cnp}} \]  

(10)

As the curvature \( \phi \) for the applied moment \( M_{cmp} \) is known, this can be used to determine the moment in the plate \( M_p = \phi (EI)_p \) (and the moment in the concrete element \( M_{RC} = \phi (EI)_{RC} \)). Hence from Eq. (6), the axial force in the plate \( F_p \) can be determined, which can be used to ensure that the plate remains linear elastic.
Fig. 3. Free body diagram of side plates and RC-beam.

From Eqs. (3) and (10) we can get the following relationship for \((EI)_{cmp}\) as

\[
(EI)_{cmp} = (EI)_{RC} + 2(EI)_p + 2(EA)_p h_{p,RC} h_{p,cmp} \tag{11}
\]

which is correct when all the elements remain isotropic. However to allow for cracking, the following research will be based on the flexural rigidity of the cracked plated section that assumes that the tensile strength of the concrete is zero. Previous research (Ref. 1) has shown that this rigidity gives the least scatter of results.

3. GENERIC MATHEMATICAL MODEL FOR FLEXURAL PEELING OF ANGLE PLATES GLUED TO SIDES OF RC-BEAM

3.1 Transmission of Flexural Moment \(M_p\)

Let us assume that the ends of the plate of bonded area of \((h_{p,bnd} \times h_{p,bnd})\) transmits the flexural moment \(M_p\) from the RC-beam into the plate as shown shaded in Fig. 4(a). The moment \(M_p\) is transmitted into the plate through the shear forces acting at the interface between the concrete and side plate. It will be assumed that premature debonding occurs at the corners of the square bonded area in Fig. 4(a), so that \(M_p\) is transmitted by shear in the circular bonded area in Fig. 4(b) which is shown enlarged in Fig. 5.
Fig. 4. Transmission of flexural forces through ends of side-bonded angle plates

Fig. 5. Distributing of bond stress resisting $M_p$

As a linear elastic analysis is being applied, the shear stress $\tau_h$ at a distance $h$ from the center of the bond area varies linearly and is given by

$$\tau_h = \frac{2h\tau_{\text{max}}}{h_{p,\text{brd}}}$$ (12)
where $\tau_{\text{max}}$ is the maximum shear stress at the circumference. Hence the moment increment due to the shear stress $\tau_h$ is

$$dM = 2\pi h \tau_h dh = 4\pi h^3 \tau_{\text{max}} dh / h_{p,\text{bd}}$$

and integrating over the bonded area gives

$$M_p = \int_0^{h_{p,\text{bd}}/2} dM = \frac{\pi h_{p,\text{bd}}^3 \tau_{\text{max}}}{16}$$

which gives the maximum shear stress $\tau_{\text{max}}$ as

$$\tau_{\text{max}} = \frac{16M_p}{\pi h_{p,\text{bd}}^3} = \frac{16\phi(EI)_p}{\pi h_{p,\text{bd}}^3}$$

(14)

3.2 Transmission of Axial Force $F_p$

3.2.1 Direct stress

Let us consider how the axial force $F_p$ is transmitted from the RC beam to the angle plates as shown in the plan view of the plated beam in Fig. 6(a).

![Fig. 6. Transmission of the axial forces](image)

From the equilibrium of the flexural forces in Fig. 6(a),

$$F_p e_{yy} = F_{a,x} k_{1,x} t_{p,w}$$

(14)

where $e_{yy}$ is the distance between the centroid of the angle and the concrete/plate interface as shown in Fig. 6(b). Therefore,
\[ F_{a,x} = \frac{F_{p} e_{yy}}{k_{1,x} t_{p,w}} \]  \hspace{1cm} (15)

The stress distribution across the interface is shown adjacent to the top plate in Fig. 6(b) where \( f_{a,x} \) is the maximum tensile stress and where the tensile stress is distributed over the length \((k_{2,x} t_{p,w})\) and depth of plate \(h_{p,bnd}\). As the thickness of the plate web \(t_{p,w}\) is usually much less than the width of the beam \(b_{c}\), the distribution of stress must be a function of \(t_{p,w}\) as has been shown in finite element analyses (Ref. 1). If we define the shape of the tensile stress distribution as \(s_{a}\) where the mean tensile stress is \((s_{a,x} f_{a,x})\), then

\[ F_{a,x} = (s_{a,x} f_{a,x})(k_{2,x} t_{p,w})h_{p,bnd} \]  \hspace{1cm} (16)

where \(h_{p,bnd}\) is the bonded depth of the angle plate.

Substituting Eq. (16) into Eq. (15) gives

\[ f_{a,x} = \frac{F_{p} e_{yy}}{k_{1,x} k_{2,x} s_{a,x} t_{p,w}^{2} h_{p,bnd}} \]  \hspace{1cm} (17)

and substituting Eq. (2) into Eq. (17) noting that \(k_{a,x} = (k_{1,x} k_{2,x} s_{a,x})^{-1}\) gives

\[ f_{a,x} = \frac{k_{a,x} e_{yy} h_{p,cmp} \phi (EA)_{p}}{t_{p,w}^{2} h_{p,bnd}} \]  \hspace{1cm} (18)

3.2.2 Mean shear stress

Figure 6 also shows that the axial force in the plate \(F_{p}\) must be equal to the shear force \(F_{sh,side}\) acting on the interface between the concrete and steel elements, that is

\[ F_{p} = F_{sh,side} \]  \hspace{1cm} (19)

Let us assume that \(L_{sh}\) is the effective bond length for this shear force \(F_{sh,side}\) over which an average shear stress \((\tau_{sh,side})_{m}\) is acting. It is also reasonable to assume that \(L_{sh}\) is proportional to the thickness of the plate (Ref. 1) so that \(L_{sh} = k_{sh,side} t_{p,w}\) as shown in Fig. 6. Hence, the mean shear stress \((\tau_{sh,side})_{m}\) can be written as
\begin{equation}
(\tau_{sh, side})_m = \frac{F_{sh, side}}{k_{sh, side} t_{p, w} h_{p, bnd}}
\end{equation}

From Eqs. (2), (19) and (20), we get

\begin{equation}
(\tau_{sh, side})_m = \frac{\varepsilon_p E_p A_p}{k_{sh, side} t_{p, w} h_{p, bnd}}
\end{equation}

and substituting Eq. (16) into Eq. (21) and simplifying gives

\begin{equation}
(\tau_{sh, side})_m = \frac{f_{a,x} t_{p, w}}{k_{a,x} k_{sh, side} \varepsilon_{yy}}
\end{equation}

It is worth noting that at the plate edge, the shear stress at the edge \((\tau_{sh, side})_e\) is zero as there is a free surface.

3.2.3 Interaction Between \((\tau_{sh, side})_m, \tau_{\text{max}}\) and \(f_{a,x}\)

Let us now consider the critical point at the interface where \((\tau_{sh, side})_m, \tau_{\text{max}}\) and \(f_{a,x}\) are at their maximum, which occurs at the middle of the edge of the plate end as shown in Fig. 7. If \(\tau_R\) is the resultant of \((\tau_{sh, side})_m\) and \(\tau_{\text{max}}\) in Fig. 7(b), then we have an element that is subjected to a shear stress \(\tau_R\) and compressive stress \(f_{a,x}\) as shown in Fig. 7(c).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{debonding_stresses.png}
\caption{Debonding stresses}
\end{figure}

If we assume that the debonding occurs when the principal tensile stress in Fig. 7(c) is equal to the tensile strength of the concrete, then from Mohr’s stress circle
\[-0.5 f_{a,x} + \sqrt{\tau_R^2 + (0.5 f_{a,x})^2} = f_t \quad (23)\]

The parameter \(\sqrt{\tau_R^2 + (0.5 f_{a,x})^2}\) in Eq. (23) can be written as \(\tau_R + k_f f_{a,x}\) where \(k_f = f(\tau_R, f_{a,x})\). If we assume as a first approximation that \(k_f\) is constant, then Eq. (23) becomes

\[(k_f - 0.5) f_{a,x} + \tau_R = f_t \quad (24)\]

As both the shear stress components \((\tau_{sh,side})_m\) and \(\tau_{\text{max}}\) are acting on the same plane as in Fig. 7(b), then a relationship for its resultant can be written as

\[\tau_R = \sqrt{(\tau_{sh,side})_m^2 + \tau_{\text{max}}^2} = (\tau_{sh,side})_m + k_{\text{max}} \tau_{\text{max}} \quad (25)\]

where \(k_{\text{max}} = f((\tau_{sh,side})_m, \tau_{\text{max}})\).

Similarly, we can assume \(k_{\text{max}}\) is constant as a first approximation and substituting Eq. (25) into Eq. (24) gives

\[\left(k_f - 0.5\right) f_{a,x} + \frac{f_{p,w} f_{a,x}}{k_{a,x} k_{sh,side} c_{yy}} + k_{\text{max}} \tau_{\text{max}} = f_t \quad (26)\]

and substituting Eq. (22) into Eq. (26) and simplifying gives

\[k_{a,x}^* f_{a,x} + k_{\text{max}} \tau_{\text{max}} = f_t \quad (27)\]

where \(k_{a,x}^*\) is given by

\[\left(k_f - 0.5 + \frac{f_{p,w}}{k_{a,x} k_{sh,side} c_{yy}}\right).\]

Now let us derive Eq. (27) further, in terms of the applied load, geometric and material properties of the plated beam. Substituting Eqs. (14) and (18) into Eq. (27) and noting that \(M_p = \phi (EI) p\) gives

\[\frac{k_{a,x}^* k_{a,x} c_{yy} A_p}{f_{p,w} h_{p,\text{bdt}}} + 16 k_{\text{max}} f_p \pi h_{p,\text{bdt}}^3 h_{p,\text{cmp}} = \frac{f_t}{\phi E_p h_{p,\text{cmp}}} \quad (28)\]
Furthermore, substituting Eq. (3) into Eq. (28) and \( k_{A,1} = \frac{k_{a,x}^{\#} e_{yy} A_p}{t_{p,w} h_{p,bnd}} \) (where \( k_{a,x}^{\#} = k_{a,x}^* k_{a,x} \)) and \( k_{B,1} = 16 k_{max} / \pi \) gives

\[
k_{A,1} + k_{B,1}\left(\frac{I_p}{h_{p,bnd}^3 h_{p,cmp}}\right) = \frac{f_i (EI)_{cmp}}{M_{cmp} E_p h_{p,cmp}}
\]

(29)

which can be written as the following linear variation

\[
k_{A,1} + k_{B,1} X_1 = Y_1
\]

(30)

where the variables \( X_1 = \frac{I_p}{h_{p,bnd}^3 h_{p,cmp}} \) and \( Y_1 = \frac{f_i (EI)_{cmp}}{M_{cmp} E_p h_{p,cmp}} \) are both dimensionless variables. Eq. (30) is a generic form of the mathematical model for flexural peeling of angles that bonded to sides of RC-beam.

### 3.2.4 Interaction Between \( \tau_{sh \text{, side}} \), \( \tau_{max} \) and \( f_{a,x} \)

Let us now consider that at the critical point at the edge of the interface, the edge shear stress is zero whilst \( \tau_{max} \) and \( f_{a,x} \) are at their maximum. In this case, the mean shear stress in Fig. 7(b) becomes \( \tau_{sh \text{, side}} = (\tau_{sh \text{, side}})_m = (\tau_{sh \text{, c}}) = 0 \) and resultant shear stress \( \tau_R \) in Fig. 7(c) becomes \( \tau_{max} \). Hence, Eq. (24) can be rewritten as

\[
(k_f - 0.5) f_{a,x} + \tau_{max} = f_i
\]

(31)

Using similar modifications as in previous sections, Eq. (31) can be rewritten as

\[
\frac{(k_f - 0.5) k_{a,x} e_{yy} A_p}{t_{p,w} h_{p,bnd}} + \frac{16 I_p}{\pi h_{p,bnd}^3 h_{p,cmp}} = \frac{f_i}{\phi E_p h_{p,cmp}}
\]

(32a)

or

\[
k_{A,1}^{\#} + k_{B,1}^{\#}\left(\frac{I_p}{h_{p,bnd}^3 h_{p,cmp}}\right) = \frac{f_i (EI)_{cmp}}{M_{cmp} E_p h_{p,cmp}}
\]

(32b)

where \( k_{A,1}^{\#} = \frac{(k_f - 0.5) k_{a,x} e_{yy} A_p}{t_{p,w} h_{p,bnd}} \) and \( k_{B,1}^{\#} = 16 / \pi \). It may be worth noting that when \( e_y \) in the parameter \( k_{A,1}^{\#} \) is negative, that is when the y-y axis in Fig. 6(b)
lies in the web of the angle, then \((k_f - 0.5)\) in the parameter also becomes negative so that the parameter stays positive.

It can be seen that Eq. (32b) has a similar form as Eq. (29) except for the intercept \(k_{A,1}^\#\) (because \(\tau_R = \tau_{\max}\) and \(k_{\max} = 1\) when \(\tau_{sh,c} = 0\), hence \(k_{B,1} = 16 / \pi = k_{B,1}^\#\)). This means that this equation leads to the same linear variation of the dimensionless variables \(X_I\) and \(Y_I\) as described in Eq. (30). Hence, it can be concluded that the magnitude of the mean shear stress \((\tau_{sh,sidem})_m\) does not influence the general form of the relationship between the dimensionless variables \(X_I\) and \(Y_I\) in the mathematical flexural peeling model.

3.3 Calibration with Flexural Peeling Model for Side Plates

It is worth noting that the generic equation for the angle plates glued to the sides of the RC-beams can be applied to side-plated beams. Therefore, the corresponding coefficients \(k_{A,1}\) and \(k_{B,1}\) for Eq. (29) can be obtained by comparing with an already established flexural peeling model for sides plates (Ref. 3).

Let us consider the form of the generic equation for angle plates glued to the sides of the RC-beams when applied for the glued side-plates with thickness \(t_p\) and depth \(h_p = h_{p,bnd}\). Substituting \(I_p = \frac{t_p h_p^3}{12}\), \(\epsilon_{yy} = -\frac{t_{p,w}}{2}\) and \(A_p = h_{p,bnd} t_{p,w}\) to Eq. (29) and simplifying gives

\[
k_{A,1} + \frac{k_{B,1}}{12} \left( \frac{t_p}{h_{p,cnp}} \right) = \frac{f_I (EI)_{cnp}}{M_{cnp} E_p h_{p,cnp}}
\]  

(33)

where \(k_{A,1} = -0.5k_{A,x}^{\#}\).

On the other hand, the mathematical model for side-plated glued RC-beams (Ref. 3) is

\[
k_A + k_B \left( \frac{t_p}{h_{p,cnp}} \right) = \frac{f_I (EI)_{cnp}}{M_{cnp} E_p h_{p,cnp}}
\]  

(34)

where \(k_A = 0.0185\) and \(k_B = 0.0185\). Therefore, it can be deduced from Eqs. (33) and (34) that, as preliminary approximations, \(k_{A,1} = k_A = 0.0185\) and \(k_{B,1} = 12k_B = 12 \times 0.185 = 2.220\) (Ref. 3). Substituting these values of \(k_{A,1}\) and \(k_{B,1}\) back into Eq. (29) gives
Substituting $M_{\text{cp}}$ by the moment $M_{\text{up}}$ at which flexural peeling occurs and rearranging Eq. (35) gives the following mean peeling strength

$\left( M_{\text{up}} \right)_{\text{angle}} = \frac{f_t h_{p,\text{bnd}}^3 (EI)_{\text{cmp}}}{E_p \left( 2.220 I_p + 0.0185 h_{p,\text{cmp}} h_{p,\text{bnd}}^3 \right)}$  \hspace{1cm} (36a)

where $f_t$ = Brazilian tensile strength and $(EI)_{\text{cmp}}$ = flexural rigidity of the cracked plated beam which was calculated by assuming the tensile strength of the concrete was zero.

Now let us plot all the available experimental data for side plated beams (Ref. 3) and for angle plated beams (Ref. 4) in terms of $X_1$ and $Y_1$ in Eq. (30) as shown in Fig. 8. The predicted regression line and its 95% confidence limits which had been carried out for side-plated beams data only (Ref. 3) are also shown in this figure. It can be seen from Fig. 8(a) that apart from one result the available experimental data for the angle-plated beams are reasonably close to the predicted mean regression line for the side plated beams. This means that the preliminary approximation that $k_{A,1} = k_A = 0.0185$ was reasonable even though $k_{A,1}$ is a function of $\epsilon_{yy}$ and $A_p$ of the angle plate.

The experimental results for angle plated and side plated beams are compared with Eq. 36 in Fig. 8(b). It can be seen that the mean of the results for the angle plates which occurs at 0.884 is lower than the mean for the side plates of 1.00, therefore

$\left( M_{\text{up}} \right)_{\text{angle}} = \frac{0.884 f_t h_{p,\text{bnd}}^3 (EI)_{\text{cmp}}}{E_p \left( 2.220 I_p + 0.0185 h_{p,\text{cmp}} h_{p,\text{bnd}}^3 \right)}$  \hspace{1cm} (36b)

Furthermore, the lower confidence limit occurs at 0.396 which is well below the confidence limit for the side plate of 0.805 that is shown in Fig. 8(b). Hence the characteristic peeling strength of angle plated beams can be taken as

$\left( M_{\text{up}} \right)_{\text{angle, side}} = 0.4 \left( M_{\text{up}} \right)_{\text{angle}} = \frac{0.4 f_t h_{p,\text{bnd}}^3 (EI)_{\text{cmp}}}{E_p \left( 2.220 I_p + 0.0185 h_{p,\text{cmp}} h_{p,\text{bnd}}^3 \right)}$  \hspace{1cm} (37)
Fig. 8a. Calibration of generic flexural peeling model for angle plate

Fig. 8b. Scatter of results
4. GENERIC MATHEMATICAL MODEL FOR FLEXURAL PEELING OF ANGLE PLATES GLUED TO SOFFIT OF RC-BEAM

4.1 Transmission of Axial Force $F_p$

4.1.1 Direct stress

Let us consider how the axial force $F_p$ is transmitted from the RC beam to the angle plates as shown in the side view of the plated beam in Fig. 9(a).

![Diagram showing transmission of axial forces](image)

**Fig. 9. Transmission of the axial forces**

From the equilibrium of the flexural forces in Fig. 9a,

$$F_p e_{xx} = F_{a,y} k_{1,y} t_{p,flk}$$

(38)

therefore,

$$F_{a,y} = \frac{F_p e_{xx}}{k_{1,y} t_{p,flk}}$$

(39)

The stress distribution across the interface is shown on the top of the side view in Fig. 9 where $f_{a,y}$ is the maximum tensile stress and where the tensile stress is distributed over the length ($k_{2,y} t_{p,flk}$) and width of the bonded area of plate $b_{p,ind}$. As the thickness of the plate $t_{p,flk}$ is usually much less than the depth of the beam $d_c$, the distribution of stress must be a function of $t_{p,flk}$ as has been shown in finite
element analyses (Ref. 1). If we define the shape of the tensile stress distribution as \( s_a \) where the mean tensile stress is \((s_{a,y} f_{a,y})\), then

\[
F_{a,y} = (s_{a,y} f_{a,y})(k_{2,y} t_{p,flk} b_{p,bnd}) \tag{40}
\]

Substituting Eq. (40) into Eq. (39) gives

\[
f_{a,y} = \frac{F_p e_{xx}}{k_{1,y} k_{2,y} s_{a,y} t_{p,flk}^2 b_{p,bnd}} \tag{41}
\]

and substituting Eq. (2) into Eq. (41) gives

\[
f_{a,y} = \frac{k_{a,y} e_{xx} b_{p,cmp} \Phi(EA)_p}{t_{p,flk}^2 b_{p,bnd}} \tag{42}
\]

where \( k_{a,y} = (k_{1,y} k_{2,y} s_{a,y})^{-1} \).

4.1.2 Mean shear stress

Figure 9 also shows that the axial force in the plate \( F_p \) must be equal to the shear force \( F_{sh,soff} \) acting on the interface between the soffit of the reinforced concrete beam and steel element, that is

\[
F_p = F_{sh,soff} \tag{43}
\]

Let us assume again that \( L_{sh} \) is the effective bond length for this shear force \( F_{sh,soff} \) over which an average shear stress \( (\tau_{sh,soff})_m \) is acting. It is also reasonable to assume that \( L_{sh} \) is proportional to the thickness of the plate (Ref. 1) so that

\[
L_{sh} = k_{sh,soff} t_{p,flk} \quad \text{as shown in Fig. 8.} \quad \text{Hence, the mean shear stress \( (\tau_{sh,soff})_m \) can be written as}
\]

\[
(\tau_{sh,soff})_m = \frac{F_{sh,soff}}{k_{sh,soff} t_{p,flk} b_{p,bnd}} \tag{44}
\]

From Eqs. (2), (43) and (44), we get

\[
(\tau_{sh,soff})_m = \frac{E_p A_p}{k_{sh,soff} t_{p,flk} b_{p,bnd}} \tag{45}
\]
and substituting Eq. (42) into Eq. (45) and simplifying gives

\[
\left( \tau_{sh,soff} \right)_{m} = \frac{f_{a,y} f_{p,ilk}}{k_{a,y} k_{sh,soff} e_{xx}} \tag{46}
\]

It is also worth noting here again that at the plate edge, the shear stress at the edge \((\tau_{sh,soff})_{e}\) is zero as there is a free surface.

4.2 Transmission of Flexural Moment \(M_p\)

The maximum peeling stress \(f_c\) in Fig. 8 that induced by the curvature \(\phi\) in the plate is determined from the equilibrium of \(M_p\) and \(F_c\) in Fig. 9 as

\[
M_p = F_c k_3 t_{p,ilk} \tag{47}
\]

where

\[
F_c = (s_c f_c) (k_4 t_{p,ilk}) b_{p,bnd} \tag{48}
\]

and \(s_c\) is the coefficient defining the shape of the stress distribution at the interface due to flexural force \(M_p\) and \((s_c f_c)\) is the mean stress.

Substituting \(M_p = (EI) \phi\) and Eq. (48) into (47)

\[
f_c = \frac{k_c \phi E_p I_p}{t_{p,ilk} b_{p,bnd}} \tag{49}
\]

where \(k_c = (s_c k_3 k_4)^{1/2}\).

4.3 Interaction Between \((\tau_{sh,soff})_{m,z}, f_{a,y}\) and \(f_c\)

Let us now consider the interaction of \((\tau_{sh,soff})_{m,z}, f_{a,y}\) and \(f_c\) at the critical point at the interface (Fig. 10(b)) which occurs at the middle of the edge of the plate end as shown in Fig. 10(a). The element that is subjected to a shear stress \((\tau_{sh,soff})_{m}\) and resultant tensile stress \(f_R = (f_{a,y} - f_c)\) as shown in Fig. 10(c).
Fig. 10. Debonding stresses

If we assume that the debonding occurs when the principal tensile stress in Fig. 10(c) is equal to the tensile strength of the concrete, then from Mohr's stress circle

\[
0.5f_R + \sqrt{(\tau_{sh,soff})^2_m + (0.5f_R)^2} = f_t
\]  

(50)

The parameter \(\sqrt{(\tau_{sh,soff})^2_m + (0.5f_R)^2}\) in Eq. (50) can be written as \(k_m(\tau_{sh,soff})_m + 0.5f_R\) where \(k_m = f ((\tau_{sh,soff})_m, f_R)\). If we assume as a first approximation that \(k_m\) is constant, then Eq. (50) becomes

\[
f_R + k_m(\tau_{sh,soff})_m = (f_c - f_{a,y}) + k_m(\tau_{sh,soff})_m = f_t
\]  

(51)

substituting Eqs. (46) and (49) into Eq. (51) and simplifying gives

\[
k^*_{a,y} f_{a,y} + f_c = f_t
\]  

(52)

where \(k^*_{a,y}\) is given by \(\frac{k_m t_p f_{tk}}{k_{a,y} k_{sh,soff} e_{xx}} - 1\).

Now let us derive Eq. (52) further, in terms of the applied load, geometric and material properties of the plated beam. Substituting Eqs. (42) and (49) into Eq. (52) and noting that \(M_p = \phi(El)_p\) gives

\[
k^*_{a,y} \left(\frac{e_{xx} A_p}{t_p f_{tk} b_p b_{ml}}\right) + k_c \left(\frac{I_p}{t_p f_{tk} b_p b_{ml} h_p h_{cmp}}\right) = \frac{f_t}{\phi E_p h_p h_{cmp}}
\]  

(53)
where \( k_\alpha^\# = k_{\alpha,y} k_{\alpha,y}^* \).

Furthermore, substituting Eq. (3) into Eq. (53) and

\[ k_{A,2} = k_{\alpha,y} \left( \frac{e_{xx} A_p}{t_p^2 b_{p,bhd} h_{p,cmp}} \right) \]

and

\[ k_{B,2} = k_c \]

gives

\[ k_{A,2} + k_{B,2} \left( \frac{I_p}{t_p^2 b_{p,bhd} h_{p,cmp}} \right) = \frac{f_t (EI)_{cmp}}{M_{cmp} E_p h_{p,cmp}} \]  \( (54) \)

which can be written as the following linear variation

\[ k_{A,2} + k_{B,2} X_2 = Y_2 \]  \( (55) \)

where the variables \( X_2 = \frac{I_p}{t_p^2 b_{p,bhd} h_{p,cmp}} \) and \( Y_2 = \frac{f_t (EI)_{cmp}}{M_{cmp} E_p h_{p,cmp}} = \frac{f_t}{E_p} \) are both dimensionless variables. Equation (54) is a generic form of the mathematical model for flexural peeling of angles that bonded to sides of RC-beam.

4.4 Interaction Between \((\tau_{sh,soft})_c\) and \(f_R\)

Let us now consider that at the critical point at the edge of the interface, the edge shear stress is zero whilst \( f_R \) are at their maximum. In this case, the mean shear stress in Fig. 10(b) becomes \( (\tau_{sh,soft})_m = (\tau_{sh,soft})_e = 0 \). Hence, Eq. (51) can be rewritten as

\[ f_R = -f_{a,y} + f_c = f_t \]  \( (56) \)

Using similar modifications as in previous sections, Eq. (56) can be rewritten as

\[ k_{A,2}^\# + k_{B,2} \left( \frac{I_p}{t_p^2 b_{p,bhd} h_{p,cmp}} \right) = \frac{f_t (EI)_{cmp}}{M_{cmp} E_p h_{p,cmp}} \]  \( (57) \)

where \( k_{A,2}^\# = -k_{a,y} \left( \frac{e_{xx} A_p}{t_p^2 b_{p,bhd}} \right) \) and \( k_{B,2} = k_c \).
It can be seen that Eq. (57) has similar form as Eq. (54) except for the intercept \( k_{A,2}^\# \). This means that this equation leads to the same linear variation of the dimensionless variables \( X_2 \) and \( Y_2 \) as described in Eq. (54). Hence, it can be concluded that the magnitude of the mean shear stress \( (\tau_{sh,soft})_m \) does not influence the general form of the relationship between the dimensionless variables \( X_2 \) and \( Y_2 \) in the mathematical flexural peeling model.

**4. 5 Calibration with Flexural Peeling Model for Soffit Plates**

Using a similar argument as has been mentioned in the section 3.4, the generic flexural peeling equation for the angle plates glued to the soffit of the RC-beam would be valid for the soffit-plated beams as well. Based on this assumption and using the available flexural peeling experimental data (Ref. 1), the coefficients of the Eq. (54) can be obtained.

Let us consider a RC-beam with steel plates of thickness \( t_p = t_{p,flk} \) and width \( b_p = b_{p,bnd} \) which are glued to the soffit of the beam. Substituting \( l_p = \frac{b_{p,bnd}t_{p,flk}^3}{12} \),

\[ A_p = b_{p,bnd}t_{p,flk} \quad \text{and} \quad e_{xx} = -\frac{t_{p,flk}}{2} \] to Eq. (53) and simplifying gives

\[
-k_{a,2}^\# + \frac{k_{B,2} R^2}{24} \left( \frac{t_{p,flk}}{h_{p,cmp}} \right) = -\frac{f_i(EI)_{cmp}}{M_{cmp}E_p h_{p,cmp}}
\]

or

\[
k_{A,2} + \frac{k_{B,2} R^2}{12} X_2 = Y_2
\]

The values of \( k_{A,2} \) and \( k_{B,2} \) in Eq. (59) can be obtained by calibration with the 57 available experimental flexural peeling data, details of which are reported elsewhere (Ref. 1).

It is again worth noting here that in deriving the variables \( X_2 \) and \( Y_2 \) in Eq. (59), \( M_{cmp} \) was the moment \( M_{up} \) at which flexural peeling occurs, \( f_i \) was the Brazilian tensile strength and \( (EI)_{cmp} \) is the flexural rigidity of the cracked plated beam which was calculated by assuming the tensile strength of the concrete was zero. The results of the regression analysis are shown in Fig. 11.

A linear regression analysis of the available data in Fig. 11 gave the intercept \( k_{A,2} = -0.00124 \) and slope \( k_{B,2} / 12 = 0.4845 \) ie.
\( k_{B,2} = 12 \times 0.4845 = 5.814 \) and the standard deviation \( STDV=0.0102 \). Substituting these values of \( k_{A,2} \) and \( k_{B,2} \) into Eqs. (54) and (59) and rearranging give the following mean peeling strength for angle plates glued to the soffit of RC-beam

\[
(M_{up})_{m,\text{soff}}^{\text{angle}} = \frac{f_{t}^2 f_{p,\text{bnd}}^2 (EI)_{\text{cmp}}}{E_p (5.814 t_p - 0.0012 t_{p,\text{bnd}}^2 h_{p,\text{cmp}})}
\] (60)

and mean peeling strength for soffit plates as

\[
(M_{up})_{m,\text{soff}}^{\text{plate}} = \frac{f_{t} (EI)_{\text{cmp}}}{E_p (0.485 t_p - 0.0012 h_{p,\text{cmp}})}
\] (61)

![Graph](image)

Fig. 11. Calibration of the generic flexural peeling model for soffit plates

It is worth noting that Eq. (61) obtained in this study is consistent with the prediction obtained by Oehlers and Moran (Ref.1) which is as follows

\[
(M_{up})_{m,\text{soff}}^{\text{plate}} = \frac{f_{t} (EI)_{\text{cmp}}}{E_p (0.474 t_p + 0.0083 h_{p,\text{cmp}})}
\] (62)

Furthermore, Eqs. (61) and (62) would give very close results because the terms \(-0.0012 h_{p,\text{cmp}} \) and \(0.0083 h_{p,\text{cmp}} \) are of very little significant compared to \(0.485 t_p\)
and 0.474t_p respectively. The terms -0.0012h_{p,cmp} and 0.0083h_{p,cmp} are derived from the transmission of the axial force F_p in Section 4.1.1, and as these terms tend to zero, this component can be ignored so that the beneficial effect of the compressive stress f_{a,y} is lost probably due to flexural cracking adjacent to the plate end.

From Eq. 62 and Ref. 1 where the characteristic strength is 52.6% of the mean strength, the characteristic peeling strength for tension face plated beams is

\[
(M_{up})_{ch,sp}^{plate} = 0.526 \times \frac{f_t(EI)_{cmp}}{E_p 0.474t_p} = \frac{f_t(EI)_{cmp}}{0.901E_p t_p}
\]

(63)

and, hence, the characteristic strength of the angle plate glued to the tension face only is

\[
(M_{up})_{ch,sp}^{angle} = \frac{0.526 f_t^2 p_{p, fk} b_{p, bnd}(EI)_{cmp}}{E_p 5.688 t_p} = \frac{f_t^2 p_{p, fk} b_{p, bnd}(EI)_{cmp}}{10.81E_p t_p}
\]

(64)

where the coefficient 5.688 = 12 \times 0.474.

5. DESIGN APPROACH TO PREVENT PEELING OF ANGLE PLATES

5.1 Comparison Between Experimental and Numerical Results

Let us now consider what model would provide the reasonable prediction compared with the available flexural peeling experimental data (Ref. 4). Table 1 shows the details of the geometry, material properties of the cross-sections of the angle plated RC-beams and the test results (Ref. 4). The average tensile strength of the concrete is f_t = 4.35 MPa and E_p = 2.05E5 MPa.

Table 1. Geometry, material properties of the cross-section of angle plated RC-beams and the test results (Ref. 4)

<table>
<thead>
<tr>
<th>#</th>
<th>Angle</th>
<th>f_{c,est} (MPa)</th>
<th>h_{p,bnd} (mm)</th>
<th>b_{p,bnd} (mm)</th>
<th>I_p (mm^4)</th>
<th>h_{p,cmp} (mm)</th>
<th>(EI)_{cmp} (Nmm^2)</th>
<th>(M_p)_{cm}^{sp} (kNmm)</th>
<th>(M_p)_{cm}^{sp} (kNmm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4a</td>
<td>150x90x8N</td>
<td>4.34</td>
<td>142</td>
<td>82</td>
<td>4391578</td>
<td>101.3</td>
<td>2.17E13</td>
<td>84.8</td>
<td>0.092</td>
</tr>
<tr>
<td></td>
<td>150x90x8S</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4b</td>
<td>75x100x8N</td>
<td>4.37</td>
<td>67</td>
<td>92</td>
<td>656012</td>
<td>134.2</td>
<td>2.19E13</td>
<td>62.3</td>
<td>0.704</td>
</tr>
<tr>
<td></td>
<td>75x100x8S</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4c</td>
<td>150x45x8N</td>
<td>4.37</td>
<td>142</td>
<td>37</td>
<td>3448479</td>
<td>99.3</td>
<td>1.7E13</td>
<td>78.5</td>
<td>0.042</td>
</tr>
<tr>
<td></td>
<td>150x45x8S</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
</tr>
</tbody>
</table>

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It can be seen from Table 1 that the generic model for angle plates glued to the sides of the RC-beam would control the peeling strengths of the angle-plated beam because its predicted values are significantly higher than those predicted by the other generic model for angle plates glued to the soffit of the RC-beam. However, more experimental data are required to confirm this conclusion.

5.2 Design Approach for Glued Angle Plates

Based on the conclusions obtained from the comparison between the available experimental and numerical results carried out in section 5.1, the following approach is recommended for the design of angle-plated RC-beams:

(i) Calculate the mean flexural peeling strength of the angle-plated RC-beam using both proposed models given by Eqs. (36) and (60).
(ii) The model which gives the higher flexural peeling strength of the angle-plated RC-beam would control the design.
(iii) Characteristic value of the flexural peeling strength predicted by the would-be-used model in step (ii) could be used for safety reason.

6. CONCLUSIONS

Two generic mathematical models have been developed for predicting the ultimate flexural peeling strength of angle plates glued to RC-beams which show good correlation with experimental data. However, the generic model for angle plates glued to the sides of the RC-beam would control the peeling strengths of the angle-plated beam. As a result, a rational design approach for designing of the angle-plated glued RC-beams was recommended for practice.

The ultimate flexural peeling strength was found to be directly dependent on the tensile strength of the concrete and the flexural rigidity of the cracked plated section. However, it was found to be inversely proportional to elastic modulus of the angle. Furthermore, the ultimate flexural peeling strength was found to be increased as the ratio of the moment of the inertia of the angle to the cubic of the depth of side-bonded area of the angle or the distance between neutral axes of the angle plates and composite cracked angle-plated section decreased.
7. ACKNOWLEDGMENTS

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8. REFERENCES


9. APPENDIX: NOTATION

A = area
b = width
d = depth
dh = length increment
E = Young modulus
e = distance from centroidal axis to the inner surface of the angle cross-section
F = force
f = material strength
h = vertical distance
I = second moment of inertia
k = proportional constant
L = longitudinal distance
M = moment
N.A = neutral axis
$s_0, f_a$ = mean tensile stress at the concrete/plate interface
STDV = standard deviation
t = thickness
X,Y = dimensionless variables
$\varepsilon$ = strain
$\phi$ = curvature
$\tau$ = shear stress

*Suffixes*

a = axial
bnd = bonded area
c = concrete
ch = characteristic
cmp = composite section
e = edge
flk = flank
m = mean
p = plate
R = resultant
RC = reinforced concrete
sh = shear
soff = soffit
t = resultant tensile
up = ultimate peeling moment
exp = experimental
w = web
xx, yy = centroidal axes of the angle cross-section