Analysis of the Behaviour and Collapse of Concrete Frames Subjected to Severe Ground Motion

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Abstract
A finite-element based analysis is described which models the behaviour and collapse of concrete plane frames when subjected to severe ground motion. Full account is taken of geometric and material non-linearities in the frame, as well as damping. While the method can be applied to multi-storey and multi-bay frames, the case of a simple portal frame is used here in a detailed numerical study. Implications of the results of the numerical study are discussed, and some limitations of the method of analysis are pointed out. Details of current extensions to the work are mentioned briefly. These include an experimental study and the modelling of the overload behaviour of high-moment high-shear hinge regions.

Key words
reinforced concrete, structural frames, earthquake, ground motion, collapse, overload behaviour, non-linear analysis,
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1. Introduction

This report presents preliminary results from an ongoing investigation into the collapse behaviour of concrete frames when subjected to severe ground motion. The first aim of the investigation is to develop a reliable and accurate computer-based analysis method which simulates the non-linear behaviour and collapse of concrete frames when subjected to dynamic loads, including ground motion. The second aim is to use the method in parametric studies to investigate the adequacy of current earthquake design methods which are based on linear and linearised analysis. The final, main aim is to derive global safety coefficients which can be used in the direct non-linear collapse design of concrete framed structures subjected to earthquake loads.

In the preliminary work described here, a non-linear analysis and computer-based simulation program have been developed for flexural frames subjected to dynamic loading and ground motion. This work is an extension and adaptation of previous studies of the time-dependent static behaviour of frames under the effects of creep and shrinkage, and of collapse under static overload. In order to test the numerical stability and modelling adequacy of the theoretical work, the behaviour of two simple portal frames has been studied in some detail for a variety of static and dynamic load conditions. The results of this numerical study are used to evaluate the analytic method and hence to identify areas where further work is needed.

2. Non-linear analysis methods for concrete structures

The accurate analysis of the behaviour of concrete structures at working load and at collapse became feasible a generation ago with the development of high-speed computing facilities. Only with computer assistance is it possible to undertake the very large number of linear analyses which are required in the iterative search methods used to evaluate the non-linear behaviour of concrete structures. Non-linearities occur as a result of both the behaviour of the component materials and the relatively large displacements which occur as a slender frame approaches collapse. Even with the very fast computing facilities currently available, highly efficient numerical procedures are needed to allow the non-linear analysis of complex concrete structures to be undertaken in reasonable time.
Most of the non-linear structural analysis procedures currently in use rely on three basic concepts, namely discretisation, automated linear structural analysis and automated, iterative search.

The discretisation of the problem is undertaken in space and time. Spatial discretisation is achieved by replacing the continuous structural system by members (such as beams, columns, walls, slabs, etc.) and the members by a finite number of segments, or elements, which are joined at a finite number of node points. In the simple case of a structural frame with flexural members (beams and columns) the cross-sections of the element are further broken into thin layers of material. The state of the structural system can then be represented by the state of stress and strain in a finite number of fibres of steel and concrete, and by the displacements of a finite number of node points.

In the dynamic analysis of the time-varying behaviour of the system, only a finite number of time points is considered within the time interval of interest, and the state of the structural system is determined at these time points. In the case of static loading a series of small load increments is considered, so that the state of the system is determined at a finite number of load levels.

In order to determine the state of the system at a particular load point or time point an automated, iterative search procedure is employed to find the nodal displacements and stresses and strains which satisfy all local and overall requirements of equilibrium and compatibility, as well as the stress-strain relations for the individual material fibres. The search is undertaken using linear analysis of the structural system with equivalent linear properties of the material fibres.

The development of efficient, automated methods of linear structural analysis in the 1950s and 1960s was a major step forward which made possible non-linear structural analysis. Historical information on the development of matrix methods of structural analysis, with a bibliography of early literature, is provided by Argyris (1958) and by Przemieniecki (1985).

Following the introduction of automated methods of linear structural analysis, the development of non-linear methods to take account of complex material response and large geometric displacements was rapid. The current approaches to non-linear analysis are described in various texts, including Zienkiewicz and Taylor (1993), Cook et al. (1989); and Kanchi (1993). Variants of the standard approaches have been proposed (Spacone, et al., 1994; Miramontes, et al., 1996; Marur & Kant, 1994). The standard approach to non-linear dynamic structural analysis which is used in this report is clearly explained by Song and Maekawa (1991).

Non-linear structural analysis has now reached the stage of development where commercial packages for structural analysis usually include non-linear options,
for example for the treatment of geometric non-linearities. While comprehensive and versatile non-linear finite element packages are also starting to appear on the market, such as DIANA (Anonymous, 1996) and MARC (http) their application to collapse load analysis is by no means a routine matter. Nevertheless, the dividing line between current research and design application of large-scale computation packages is blurred.

A central step in the development of any body of theoretical analysis, and also in the development of reliable computer simulation packages for practical application, is the comparison of theoretical predictions with real experimental data. While large-scale and small-scale static tests of structural systems can be undertaken with relative ease in well-equipped laboratories, it is difficult to obtain true dynamic test data for concrete structural systems. Lack of confirmation of non-linear structural analysis programs is currently one of the ongoing problems of researchers and practitioners.

3. Details of present method of analysis

3.1 Overview

The analysis considers the time-varying behaviour and collapse of a plane reinforced concrete frame (Fig 1) when subjected simultaneously to a sustained gravity load system and severe ground motion. It was developed by extending an existing non-linear static analysis method for frames (Kawano & Warner, 1995) to take account of inertial and damping effects.

The single-mass system shown in Fig 2 provides a simple starting point to discuss the dynamic analysis of frames. The mass is subjected to a horizontal force $F(t)$, and, although restrained by the spring and dashpot which are attached to the base, can undergo horizontal movement $y(t)$ relative to the base. The base motion $y_E(t)$ and the applied force $F(t)$ are considered to be known functions of time. The base motion $y_E(t)$ and the applied force $F(t)$ are considered to be known functions of time. The force in the spring depends on the displacement of the mass relative to the base, $y(t)$, and is written as $P(y(t))$. The force in the dashpot depends on the rate of change of $y$, i.e. $\dot{y}(t)$, and hence is written as $Q(\dot{y}(t))$. The inertial force acting on the mass depends on its acceleration, which is $(\ddot{y}_E(t) + \ddot{y}(t))$. The equation of motion for the mass at time $t$ is thus:

\[ F(t) - P(y(t)) - Q(\dot{y}(t)) = m(\ddot{y}_E(t) + \ddot{y}(t)) \] \hspace{1cm} (1)

In the simple case of a linear spring with constant stiffness $K$ and linear damping with viscoelastic constant $C$, the equation becomes:

\[ F(t) - Ky(t) - C\dot{y}(t) = m(\ddot{y}_E(t) + \ddot{y}(t)) \] \hspace{1cm} (1a)
In order to write corresponding equations of motion for the frame in Fig 1 it is first represented approximately by a discretised model. Each component member of the frame (beams and columns) is replaced by a number of beam finite elements which meet at node points, while the distributed mass of the member is approximated by concentrated masses located at the nodes. Generally, the node points can move horizontally and vertically, and external vertical and horizontal forces may act at the nodes. The vector of nodal displacements is written as \( y(t) \) and the vector of imposed nodal forces as \( F(t) \). The adjoining beam elements exert forces on the nodes and the magnitudes of these forces depend on the nodal displacements. The vector of the forces is therefore written as \( P(y(t)) \). Finally, viscous damping is represented by a vector of forces acting at the nodes, \( Q(t) \). The equations of motion for the nodal masses \( M \) can be expressed in vector form which is comparable with Eq 1:

\[
F(t) - P(y(t)) - Q(\ddot{y}(t)) = M[\ddot{y}(t) + \dot{y}(t)]
\]  

(2)

In the case of a simple linear elastic structural system where \( K \) is constant, the force vector \( P(y(t)) \) is related to the nodal displacements \( y(t) \) by the stiffness matrix \( K \) of the system:

\[
P(t) = Ky(t)
\]  

(3)

In the more general situation considered here, the structure may experience large internal deformations and large nodal displacements which increase exponentially as collapse is approached. The resulting material and geometric non-linearities are taken into account in the dynamic analysis in the same way as in a static analysis. In particular, geometric non-linearities are treated using an Updated-Lagrangian formulation with a local coordinate axis for each element which moves with the element within the global axis system. To treat material non-linearities, the cross-sections of the elements are discretised into a large number of thin 'fibres' of concrete and steel reinforcement, and appropriate constitutive relations are applied to these fibres. Provision is thus made for yielding of the steel reinforcement, and inelastic behaviour of the concrete, including post-peak softening. The constitutive relations allow for loading-unloading cycles in both materials, so that hysteretic damping is taken into account. It also follows that the force vector \( P(t) \) depends on the current nodal displacements \( y(t) \) and also on the history of previous displacements if the frame has already experienced overload. To deal with the time-dependencies and path-dependencies of the problem, we restrict attention to a finite number of time instants \( t_0, t_1, \ldots, t_{n-1}, t_n, \ldots \), with a typical time increment between successive instants of:

\[
\Delta t_n = t_n - t_{n-1}.
\]  

(4)
The time-varying response of the structure is determined at these pre-chosen time instants. For each time point, an iterative computational cycle is employed to obtain the state of stress and strain in each layer of material in every element, and the state of the overall system, as represented by nodal displacements and stress resultants. The changes which occur during the time interval $\Delta t_n$ are determined iteratively, taking account of the conditions at $t_{n-1}$, so that the state at the next time instant $t_n$ can be obtained. This procedure is an extension of the static non-linear frame analysis. Before discussing the details of the computation cycle for the dynamic analysis, it is therefore useful to describe briefly the method for non-linear static analysis, which provides the basis for the dynamic analysis.

### 3.2 Non-linear static frame analysis

The static analysis is carried out for a sequence of load steps, using small prescribed increments. The method has been described elsewhere (Kawano & Warner, 1995) and has been used to investigate the slow (non-inertial) time-varying behaviour of concrete structures induced by creep and shrinkage effects. We consider here the computations for a typical short-term load increment $\Delta P_n$ which takes the load from $P_{n-1}$ to $P_n$. The state of the system at load $P_{n-1}$, including the nodal displacements $y_{n-1}$, has been determined in the previous computation step. The computations used to determine the new displacements $y_n$ and the new state of the structural system begin with several preliminary steps which are followed by an iterative cycle. The procedure uses the Newton-Raphson method, which is illustrated for a simple one-dimensional situation in Fig 3.

Firstly, the tangent modulus for each material fibre is determined using the known conditions at the previous load $P_{n-1}$. The nodal displacements $y_{n-1}$ allow the strain in each fibre in each element to be obtained, and the constitutive relations then give the corresponding stress and hence the fibre tangent stiffness. This in turn allows the tangent stiffnesses of the elements and of the overall system to be evaluated. The system tangent stiffness matrix at $y_{n-1}$ is written as $K_t$. Using the known load increment $\Delta P_n$ and $K_t$, a linear analysis is carried out to obtain the first estimate of the increment in the displacement vector, $\Delta y_n$:

$$\Delta P_n = 0K_t \Delta y_n$$  \hspace{1cm} (5)

The scalar equivalent of this step can be seen in Fig 3. The first estimate of the new displacement vector is thus

$$0y_n = y_{n-1} + 0\Delta y_n.$$  \hspace{1cm} (6)
The corresponding strains throughout each element can be determined from these nodal displacements, and hence the fibre stresses from the constitutive relations. This allows the nodal force vector, \( \mathbf{0P}_n \), which correspond to \( \mathbf{0y}_n \), to be evaluated. The difference between the vector of specified nodal forces \( \mathbf{P}_n \) and this initial estimate, \( \mathbf{0P}_n \), is the vector of unbalanced nodal forces \( \mathbf{0O}_n \):

\[
\mathbf{0O}_n = \mathbf{P}_n - \mathbf{0P}_n
\]  

(7)

which, in effect, is a vector of error terms. If \( \mathbf{0O}_n \) is very small the computational cycle is considered to be complete, and the value of the displacement vector \( \mathbf{0y}_n \) is accepted as \( \mathbf{y}_n \). The state of the system can then be recorded before proceeding to the next load increment.

Otherwise, a sequence of iterations is undertaken, commencing with \( i = 1 \), and continuing, \( i = i + 1 \), to improve the current estimate of the displacement vector, \( \mathbf{y}_n \), until the error \( \mathbf{0O}_n \) is negligible. In the \( i \)-th iteration a new value for the tangent modulus, \( \mathbf{K}_t \), is calculated using the previous estimate of nodal displacements \( \mathbf{0y}_n \). Thus, if \( i = 1 \), \( \mathbf{K}_t \) is calculated from \( \mathbf{0y}_n \). A correction term \( \mathbf{iO}_n \) is obtained from the following equation:

\[
\mathbf{iO}_n = \mathbf{K}_t \mathbf{i\delta y}_n
\]  

(8)

so that the new estimate for \( \mathbf{y}_n \) is:

\[
\mathbf{iy}_n = \mathbf{iO}_n + \mathbf{i\delta y}_n
\]  

(9)

Using \( \mathbf{iy}_n \) a new set of nodal forces is determined and hence a new vector of unbalanced forces \( \mathbf{iO}_n \). The iterations continue until the error term is negligible. The mathematical details of this analysis are presented in Appendix B of this report.

In this analysis, the application of load is treated in a manner analogous to load-control testing in the laboratory. In some circumstances it is preferable to increment the displacements \( \Delta \mathbf{y} \) in the manner of deformation-control testing in the laboratory. Some simple modifications can be made to allow for this variation, but they are not discussed here.

3.3 Dynamic analysis

The step-by-step calculation procedure used for the dynamic analysis is a modification of that described in the previous section for static analysis. Time
steps are considered in lieu of load steps so that the computational cycle for the
time interval $\Delta t_n = t_n - t_{n-1}$ replaces the static load step for $\Delta P_n$. The conditions at
time $t_{n-1}$ are completely known, having either been determined in the previous
computation cycle, or, for the first time increment, specified by the initial
conditions.

Applying Eq 2 to the time instants at the beginning and at the end of the time
increment, we have:

$$F(t_{n-1}) - M\{\ddot{y}_E(t_{n-1}) + \ddot{y}(t_{n-1})\} - C(\dot{y}(t_{n-1})) - P(y(t_{n-1})) = 0 \quad (10)$$

$$F(t_n) - M\{\ddot{y}(t_n) + \ddot{y}(t_{n-1})\} - C(\dot{y}(t_n)) - P(y(t_n)) = 0 \quad (11)$$

Subtracting Eq 10 from Eq 11 and introducing the following definitions:

$$\Delta y = y(t_n) - y(t_{n-1}) \quad (12)$$

$$\Delta \dot{y} = \dot{y}(t_n) - \dot{y}(t_{n-1}) \quad (13)$$

$$\Delta F = F(t_n) - F(t_{n-1}) \quad (14)$$

$$\Delta \ddot{y}_E = \ddot{y}_E(t_n) - \ddot{y}_E(t_{n-1}) \quad (15)$$

$$\Delta \ddot{y} = \ddot{y}(t_n) - \ddot{y}(t_{n-1}) \quad (16)$$

we obtain, for the time interval $\Delta t_n$:

$$\Delta F - M\{\Delta \ddot{y}_E + \Delta \ddot{y}\} - C(\Delta \dot{y}) - \Delta P(\Delta y) = 0 \quad (17)$$

The term $\Delta P(\Delta y)$ in this equation is the increment in the nodal force vector
which occurs during $\Delta t_n$, and depends on the vector of displacement increments
$\Delta y$. The system tangent stiffness can be used to relate $\Delta y$ to $\Delta P$:

$$\Delta P = K_t \Delta y \quad (18)$$

The tangent stiffness matrix $K_t$ is made up of contributions from each element in
the frame. The contribution of an element consists of three components:

- a linear component which represents the tangent stiffness at the current
  stress levels;
- a non-linear component representing rigid body motion of the element; and
- a non-linear component representing displacements within the length of the element.

From Eq 18 we can substitute for \( \Delta \mathbf{P} \) into Eq 17 to give:

\[
\Delta \mathbf{F} - \mathbf{M} \{ \Delta \dot{y}_E + \Delta \dot{y} \} - \mathbf{C} \Delta \ddot{y} = \mathbf{K}_t \Delta \mathbf{y} 
\]

(19)

To solve for \( \Delta \mathbf{y} \), expressions are needed for the increments in the first and second derivatives of \( y \). Estimates of these quantities can be obtained using the Newmark method (Newmark, 1959) as follows:

\[
\Delta \dot{y} = \frac{1}{2 \beta \Delta t} \Delta \mathbf{y} - \frac{1}{2 \beta} \dot{y}_{n-1} + \left[ \frac{\Delta t}{4 \beta} - \frac{\Delta \ddot{y}}{4 \beta} \right] \dot{y}_{n-1} 
\]

(20)

\[
\Delta \ddot{y} = \frac{1}{\beta \Delta t^2} \Delta \dot{y} - \frac{1}{\beta \Delta t} \ddot{y}_{n-1} - \frac{1}{2 \beta} \dddot{y}_{n-1} 
\]

(21)

Equations 20 and 21 allow the increments in the derivatives to be estimated.

Substitution of these equations into Eq 19 and rearrangement leads to the following:

\[
\mathbf{K}_t^* \Delta \mathbf{y} - \Delta \mathbf{F}^* = 0 
\]

(22)

where

\[
\mathbf{K}_t^* = \mathbf{K}_t + \frac{1}{2 \beta \Delta t} \mathbf{C} + \frac{1}{\beta \Delta t^2} \mathbf{M} 
\]

(23)

\[
\Delta \mathbf{F}^* = \Delta \mathbf{F} + \mathbf{M} \{ -\Delta \dot{y}_E + \frac{1}{2 \beta} \dot{y}_{n-1} + \frac{1}{\beta \Delta t} \ddot{y}_{n-1} \} + \mathbf{C} \{ \frac{1}{2 \beta} \dddot{y}_{n-1} + \left( \frac{\Delta t}{4 \beta} - \Delta \ddot{y} \right) \dot{y}_{n-1} \} 
\]

(24)

The first estimates of the vector increment \( \Delta \mathbf{y} \) is obtained by solving Eq 22 and then used to determine the first estimate of \( \mathbf{y}_n \):

\[
\mathbf{y}_n = \mathbf{y}_{n-1} + \Delta \mathbf{y} 
\]

(25)

Using this first estimate of the nodal displacements, updated values of \( \Delta \dot{y} \) and \( \Delta \ddot{y} \) can be obtained from Eqs 20 and 21, and hence for \( \Delta \mathbf{y} \). Updated values for the nodal forces \( \mathbf{P}(\Delta \mathbf{y}) \) can also be obtained from \( \Delta \mathbf{y} \). This allows a check on the unbalanced nodal forces at time \( t_n \) to be made:

\[
\mathbf{O}_n = \mathbf{F}_n - \mathbf{M} \{ \mathbf{F}_n + \Delta \mathbf{y} \} - \mathbf{C} \Delta \mathbf{y} - \mathbf{P}(\Delta \mathbf{y}) 
\]

(26)

If \( \Delta \mathbf{y} \) is negligible we can go on to the next time increment. However, if it is unacceptably large we commence a cycle of iterations to reduce it to an acceptably small value. In the first iteration, \( i = 1 \), and in subsequent iterations, \( i = i + 1 \), the updated tangent stiffness matrix \( \mathbf{K}_i \) is calculated using the previous
estimate of the displacement vector, \( i\hat{y}_n \). Using the current unbalanced force vector and the updated tangent stiffness matrix a correction for \( y_n \) is obtained:

\[
i \hat{O}_n = i \hat{K}_i i \hat{\delta} y_n
\]  
(27)

The updated estimate of \( y_n \) is then:

\[
i \hat{y}_n = i \hat{y}_n + i \hat{\delta} y_n.
\]  
(28)

Using \( \hat{y}_n \), estimates of the first and second derivatives can be made:

\[
i \hat{\dot{y}}_n = i \hat{\dot{y}}_n + \frac{1}{\beta \Delta t^2} \hat{\delta} y
\]  
(29)

\[
i \hat{\ddot{y}}_n = i \hat{\ddot{y}}_n - \frac{1}{2\beta \Delta t} \hat{\delta} y
\]  
(30)

Using the new estimate of the nodal displacements, the local strains can be determined, and hence the stresses, in each fibre in each element. The new nodal force vector can then be obtained and hence an updated error vector \( i \hat{O}_n \). The iterations continue until convergence is achieved.

### 3.4 Computational details

In transforming from generalised stresses \( s \) in the beam element to end forces \( f \), a numerical integration is required. This is represented mathematically by Eq 38 in Appendix B, and is achieved computationally using three-point Gaussian integration. The integration points are located at the mid point of the element and at two points close to each end. A cross-sectional analysis is undertaken at each Gauss point for the moment and axial force acting, and using the discretised cross-section of the element with layers of concrete and steel.

In applying Newmark's method (Eqs 20 and 21 in Section 3.3 above) the value of the parameter \( \beta \) is taken as 0.25. This corresponds to an assumption of constant acceleration during each time step \( \Delta t \), and guarantees numerical stability.

At each time step the Newton-Raphson procedure described in Section 3.3 was used to determine the equilibrium state. However, if convergence was not achieved within a pre-specified number of iteration cycles the calculations for the step were automatically discontinued and recommenced with a time interval automatically reduced by fifty per cent. Provision was made for a predetermined number of interval reductions. If convergence was not achieved within the prescribed number of repetitions then the computations were discontinued with a non-convergence message.

The Rayleigh damping method is assumed for computational purposes. In this method, the frame damping is specified using damping factors for two vibration
modes. The system damping matrix is assembled using a weighted summing of
the stiffness matrix and the mass matrix.

3.5 Stress-strain relationships for concrete and steel

The model by Popovics (Popovics 1973 and Mander et al. 1988) is adopted here
as the instantaneous stress-strain model for concrete.

For compression:

\[ \sigma = f_c \cdot \frac{r_c \left[ \varepsilon_{ep} / \varepsilon_{pc} \right]_c}{r_c - 1 + \left| \varepsilon_{ep} / \varepsilon_{pc} \right|_c} \]

\[ r_c = \frac{E_c}{E_c - E_{sc}}, \hspace{1cm} E_{sc} = f_c / \varepsilon_{pc} \]

For tension:

\[ \sigma = f_t \cdot \frac{r_t \left[ \varepsilon_{ep} / \varepsilon_{pt} \right]_t}{r_t - 1 + \left| \varepsilon_{ep} / \varepsilon_{pt} \right|_t} \]

\[ r_t = \frac{E_c}{E_c - E_{st}}, \hspace{1cm} E_{st} = f_t / \varepsilon_{pt} \]

where \( f_c \) is the compressive strength and \( f_t \) is the tensile strength. The strains \( \varepsilon_{pc} \)
and \( \varepsilon_{pt} \) correspond to the peak stresses in compression and tension, respectively.
\( E_{sc} \) and \( E_{st} \) are the secant moduli for compression and tension. The unloading
branch for both the compression and tension curves use the elastic modulus \( E_c \).

The model has a plateau at the compressive peak stress which expresses the
increase in deformability due to the confining effect. Mander (Mander et al.,
1988) has derived a stress-strain relationship for confined concrete by using the
full Popovics' expression. However, it is reported that the model overestimates
actual compressive strength. The present model can allow for a large
deformability without any increase in compressive strength.

The following parameters, shown in Fig 4, are used as input for the model:

- \( E \): Young's modulus
- \( f_c \): maximum stress in compression
- \( \varepsilon_{pc} \): strain at maximum stress in compression
- \( \varepsilon_{pt1} \): strain at the end of compressive plateau
\( \varepsilon_{pu} \): strain limit in compression; if strain exceeds this limit, then the stress is set and kept as zero

\( f_t \): maximum stress in tension

\( \varepsilon_{pt} \): strain at maximum stress in tension

\( \varepsilon_{ptu} \): strain limit in tension; if strain exceeds this limit, then the stress is set and kept as zero.

Typical values of the input data used in the numerical analysis in this report are as follows:

\[
E = 30,000 \text{ MPa}, \quad f_t = 35 \text{ MPa}, \quad \varepsilon_{pc} = 0.0025, \quad \varepsilon_{pc1} = 0.0025, \quad \varepsilon_{pu} = 0.1, \quad f_t = 4.0 \text{ MPa}, \quad \varepsilon_{pt} = 0.0025, \quad \varepsilon_{ptu} = 0.05
\]

The expression by Menegotto and Pinto (1973) is used for the instantaneous stress-strain relation for steel:

\[
\sigma = \sigma_y \cdot \left( \frac{[1 - b] \cdot \varepsilon / \varepsilon_y}{\left[ 1 + \frac{\varepsilon / \varepsilon_y}{R_{mp}} \right]^{1/R_{mp}}} + b \cdot \left[ \varepsilon / \varepsilon_y \right] \right)
\]

where \( \sigma_y \) is the yield stress and \( \varepsilon_y \) is the yield strain (=\( \sigma_y / E_s \)). The term \( b \) is a strain hardening ratio, \( E_{\text{hard}} \) is the tangent modulus in the strain hardening range, \( E_s \) is the Young's modulus of the steel bar. The parameter \( R_{mp} \), which characterises the rounded region of the curve, is assumed to have the values of 5.0 for the initial loading curve and 0.8 for the unloading branch to take account of the Bauschinger effect.

The following parameters, shown in Fig 5, are used as input for the model:

- \( E \): Young's modulus
- \( \tau E \): tangent modulus in strain-hardening region at initial loading
- \( \varepsilon_y \): yield strain at initial loading or within small amplitude of strains, (Note: this is a parameter for the equation given not the actual yield strain)
- \( R_{\text{ini}} \): parameter to specify the roundness of curve in initial loading
- \( \tau_2 E \): tangent modulus in strain-hardening region for proceeding loading
- \( \varepsilon_{y2} \): yield strain for proceeding loading
$R_{bsg}$: parameter to specify the roundness of hysteresis curve for proceeding loading

$R_{bsg}$: limit of strain; if strain or its amplitude exceeds this limit, then the above parameters are changed from those for initial loads to proceeding loading.

$\varepsilon_{vp}$: limit of strain amplitude; if strain amplitude is within this limit, then the reloading path goes towards the previous reverse point.

Typical numerical values of the input data used in this report are as follows:

\[ E = 200,000 \text{ MPa}, \tau E = 600 \text{ MPa}, \varepsilon_y = 0.0023, R_{ini} = 5.0, \]

\[ \tau_2 E = 400 \text{ Mpa}, \varepsilon_{y2} = 0.00215, R_{bsg} = 0.8, \varepsilon_{im} = 0.004, \varepsilon_{vp} = 0.008 \]

4. Simulation of a multi-storey framed structure under the effect of horizontal ground motion

To indicate the functioning of the computer simulation program, and its numerical stability, it was used to predict the behaviour of a multi-storey frame with constant vertical loads acting on the beams and the El Centro 1940 N-S earthquake ground motion with a maximum acceleration of 3410 mm/sec².

The frame, shown in Fig 6, has six stories and two bays. The columns in the first three floors (C1, C2 and C3) have a cross-section 800 mm square (Fig 7), and those in the upper floors (C4, C5 and C6) 700 mm square. The beams have T-sections with a total depth of 900 mm for the lower two floors (G1 and G2), and 800 mm for the upper floors (G3, G4, G5 and GR).

For the computations the masses were concentrated at the beam-column connections (Fig 9), and the vertical loads were concentrated at the middle and ends of each beam as shown in Fig 8.

The concrete strength was 30 MPa throughout, and the yield strength of the 25 mm diameter reinforcing bars 350 MPa. The discretisation of the frame for analysis, and the identification of nodes and elements are shown in Figs 10 to 12. For the cross-section analyses, eight layers of concrete were considered for both beams and columns, with three or four layers of reinforcement in the column sections and two layers for the beams.

The Popovic model was used for the concrete fibres, with an initial elastic modulus of 25,000 MPa and a strain of 0.0025 at the peak stress of 30 MPa. The Menegotto-Pinto steel model was applied to the reinforcement with an initial
elastic modulus of 200,000 MPa and a strain hardening tangent modulus of 600 MPa.

The time interval for the analysis was chosen at 0.005 seconds to achieve numerical stability. For the Newmark integrations, a value of $\beta = 0.25$ was adopted. To set up the damping matrix for the frame, Rayleigh damping was assumed with a damping factor of 0.02 for the first and sixth natural modes of vibration.

The response of the frame is summarised in Figs 13 to 19. The first six modes of motion are shown in Fig 13. The response at the various floor levels is indicated in Figs 14 to 16 for displacement, velocity and acceleration. Base shear, storey shear and energy balance are also shown in Figs 17 to 19. From Fig 14 it is clearly seen that the vibrating period of the frame structure at the end of observation is almost twice that at the beginning. This is due to the development of cracks in concrete of the structure. The cracks reduce horizontal rigidity of the structure and consequently the period of the structure increases.

5. Behaviour of simple portal frames under various static and dynamic loadings

While the results of the multistorey frame show how the analysis and program predict overall structural response to ground motion, it is difficult to interpret the quantitative results for such a complex structure. It was therefore decided to undertake a detailed numerical study of simple portal frames. The purpose was to test the capabilities of the simulation procedure, and also to predict the collapse behaviour of structures which are simple enough to allow qualitative interpretation and hence evaluation of the adequacy of the structural response predictions.

Details are given here of only two sets of simple portal frames; one set is for a stocky frame and the other for a slender frame. Each frame was used in the following analyses:

- static pushover loading;
- cycles of static push-pull loads;
- application of severe impulse ground motions, to produce frame collapse; and
- dynamic ground motions to simulate the El Centro 1940 Earthquake record.
In the case of the impulse ground motion analyses, many runs were undertaken, with varying vertical loads and various impulse details, in order to achieve conditions which resulted in dynamic collapse of the frame. Likewise, many runs were undertaken using various intensities of the earthquake ground motion. The results of typical analyses are discussed in detail below. The behaviour of the slender frame for each loading case is first considered. The somewhat simpler behaviour of the stocky frame is discussed afterwards.

5.1 Details of numerical study

The structural details of the two portal frames are shown in Fig 20. The stocky frame, with a 5 m span and column height of 1.6 m, is similar to a model frame which had been tested at the University of Adelaide (Alia, Griffith & Freeman, 1997). The slender frame is similar in detail to the stocky frame except that the length of the columns has been increased to 10 m.

The cross-section of the columns in each frame was 200 mm square with reinforcement consisting of eight Y-16 bars placed symmetrically in all four faces (three bars per face) with concrete cover of 20 mm (see Fig 20-d). The beam section was 200 mm wide and 400 mm deep, with four Y-16 bars as reinforcement in both the top and bottom faces with a clear cover of 20 mm (Fig 20-c). The reinforcement was not curtailed.

The behaviour of the concrete stress fibres was modelled using Popovic’s stress-strain relations with provision for post-peak softening in compression and tension stiffening in tension. The Menegotto-Pinto relations were used to model the steel. The concrete the strength was taken to be 35 MPa at a strain of 0.0025, with an initial modulus of elasticity of 30,000 MPa. For the steel the initial modulus of elasticity was chosen as 200,000 MPa, with a tangent stiffness of 600 MPa in the strain-hardening range. The yield stress for the steel was taken as 460 MPa.

In the discretisations used for the computer analyses of the frames each beam and column was represented by elements with an element length in all cases equal to the cross-sectional dimension of the column, ie 200 mm This limit on minimum length was chosen in order to avoid a more rigorous analysis to take into account warping (Bazant et al., 1987). The numbering of nodes shown in Fig 20 begins with node 1 at the base of the left column in each case. For the stocky frame (Fig 20-b), with eight elements per column, Node 9 is located at the top of the left column. The 5 m beam contains 25 elements so that, Node 34 is located at the right-hand end of the beam and at the top of the right column. The right hand column has Nodes 34 and 42 at its ends. In the case of the slender frame(Fig 20-a), the left hand column has Nodes 1 and 31 at its ends, and the right column has Nodes 76 and 126 at its ends.
For this simple portal frame system, the number of an element is equal to the smaller of its two-end-node numbers. For example, Element 1 always occurs at the bottom of the left column, between Nodes 1 and 2. Cross-sectional analyses were undertaken at three sections in each element for Gauss integration. For Element i, the sections were taken at 22.54 mm from Node i, at the middle of the element, and at 22.54 mm from Node i+1 as shown in Fig 20-e.

For the dynamic analyses the mass of each member was distributed proportionally to each node point. In the case of uniformly distributed loading on the beam, including self-weight, the load is assumed to act proportionally at the nodes.

For the cross-sectional analysis of the elements under the effects of moment and axial force the concrete was broken into eight layers with each being treated as a stress fibre. All levels of reinforcement in the section were treated as individual layers; for example, three layers were used for the column elements, one at the compressive and tensile face and one at mid-depth. The area of reinforcement at the outer column faces was 603.2 mm² and at the mid-depth layer 402.1 mm². For the beam, the areas of reinforcement in the top and bottom faces were 804.2 mm².

In the static load analyses, increments in load and in horizontal deflection at the beam-column connection, ie Nodes 9 (for the stocky frame) and 51, were both used at various stages. For all dynamic analyses the incremental steps were specified using small time increments which were initially chosen to be sufficiently small to satisfy the convergence criteria of Newmark’s method. At each time step the Newton-Raphson procedure described in Section 2 of this report was used to determine the equilibrium state.

In a number of the static analyses the frame was subjected to a constant vertical load (such as self-weight on the beam) and a varying horizontal load. In such situations the constant load was first applied to the frame, and then the horizontal loading was applied. This was possible because the program can accommodate non-proportional and sequential loadings.

The damping matrix for the frame was set as mass proportional damping, with a damping factor of 0.02. A value of 0.25 was adopted for the integration parameter $\beta$.

5.2 Static push-over loading to failure of slender frame

In the static push-over study, a constant distributed vertical load was first applied to the beam. The horizontal load was then applied in steps at the beam-column joint. The results of the analysis are shown in the load-deflection plot in Fig 21. In order to study the localised response of the frame to load, the stresses
and strains were considered in the cross-sections at the top and bottom of each column (Sections 2, 149, 227 and 374 as shown in Fig 21).

The key points in the load-deflection curve for the slender frame with gravity load are indicated as points 12 through 17 and point 70. These numbers correspond to the load-step. At point 12, the tension reinforcement at section 227 first reaches the nominal yield strain $\varepsilon = 0.0023$, with $P = 15.2$ kN and $\delta = 300$ mm. At point 13 ($P = 16.4$ kN and $\delta = 350$ mm) the tension reinforcement at sections 374 and 2 also exceed nominal yield, having reached strain values of 0.0027 and 0.0025, respectively. The tension steel at section 149 finally reaches its nominal yield strain at point 14 with a value of $\varepsilon_s = 0.0027$ and with $P = 17.2$ kN and $\delta = 400$ mm. Hence, the hinge-formation sequence is consistent with that predicted using simple plastic analysis. It should be noted, however, that the reinforcing steel stress does not reach its actual (average) yield stress level of $f_{y} = 460$ MPa until a strain level of approximately 0.005. It is not until point 16 that the tension reinforcement steel stress at three sections reach yield, with the stress in sections 2, 227 and 374 all reaching $f_{y} = 460$ MPa. The tensile stress at section 149 is $f_{y} = 457$ MPa at this point (where $P = 17.8$ kN and $\delta = 500$ mm). The frame attains its maximum strength at point 17 ($P = 17.9$ kN and $\delta = 550$ mm) where the tensile stress at section 149 finally reaches yield. The frame behaviour at this stage is such that it maintains its strength while it deforms a further 50 mm to point 70 ($P = 17.8$ kN and $\delta = 600$ mm) at which stage the concrete begins to crush. From this point on, the frame loses strength at an increasing rate.

The curvatures at the four key sections were also calculated for this analysis. It is worth noting that the value of curvature ($\phi_{u,\text{theory}} = 0.046$/m) corresponding to the ultimate flexural strength ($M_{u,\text{theory}} = 60$ kNm) for the column cross-section, calculated using rectangular stress block theory, compares favorably with the results of this nonlinear analysis, being $\phi_{u,\text{analysis}} = 0.047$/m and $M_{u,\text{analysis}} = 63.2$ kNm, respectively. The corresponding horizontal displacement (or “drift”) at point 17 as a percentage of the storey height, $h$, was $\delta_h = 550$ mm $= 0.055 \cdot h$. This is somewhat larger than that normally expected (or indeed permitted) in seismic design but this frame is also extremely “slender”, having columns with a slenderness ratio of $L/D = 50$

In summary, the “slender” frame with gravity loads behaved as expected. It reached its ultimate strength of $P = 17.9$ kN at a lateral storey drift of $\delta_u = 0.055 h$. The average curvature ductility for the four key sections was estimated to be $2.2 (=0.047/0.021)$ by dividing $\phi_u$ by the curvature of the cross-section ($\varphi_y = 0.021$) when the steel strain reached the nominal yield level of $\varepsilon_y = 0.0023$. In a similar fashion, the frame displacement ductility was estimated
to be 1.8 (= 550 mm / 300 mm) by dividing \( \delta_s \) (point 17) by \( \delta_u \) (point 12). These levels of ductility are quite modest and appear to be realistic. Due to the slenderness of the columns, and hence "P-\( \delta \)" effects, there is no appreciable "yield plateau" in the frame’s load-deflection curve, further highlighting the limited ductility for this system. The other curve in Fig 21 shows the force displacement curve for the case without gravity load. The difference between these curves clearly shows the pronounced effect of gravity loading on the slender frame.

5.3 Static push-pull loading of slender frame

The push-pull loading was applied statically and consisted of a deflection of 750 mm induced in one direction, and then a load reversal to achieve the same deflection in the opposite direction. This cycle of push-pull loads was repeated twice. The displacement of 750 mm was chosen as being 1.5 times the deflection achieved at peak load in the static pushover loading discussed already in Section 5.2. During the push-pull loading, no vertical sustained load was applied. For the calculations, a quasi-static reverse cyclic loading regime was used, whereby a harmonic horizontal displacement-time history was applied to the top of the frame at a very slow rate so as not to induce any dynamic response in the structure. The purpose of this analysis was to study the cyclic behaviour of the frame under extreme levels of deformation.

In Fig 22, the load-deflection plot for the slender frame subject to the static push-pull loading is shown. Of key interest was the frame behaviour around points 1 through 9 and the degree of strength and stiffness degradation displayed. On the first loading, the frame behaviour was identical to that seen for the slender frame without gravity loading in the push-over analysis. At point 1, \( P = 24.9 \) kN and \( \delta = +750 \) mm the tension steel had yielded at each of the four key sections \( (\varepsilon_{s,avg} = 0.020 = 8.5 \varepsilon_y \text{ and } f_{s,avg} = 470 \text{ MPa} > f_y = 460 \text{ MPa}) \) and the concrete had crushed on the compression face \( (\varepsilon_{c,avg} = 0.0040 > \varepsilon_{pc} = 0.0025) \). Furthermore, the middle steel had also yielded at this point \( (f_{s,avg} = 464 \text{ MPa}) \). Notably, the average curvature at the four key sections at point 1 was calculated to be \( \phi = 0.142 \text{ m}^{-1} = 3 \phi_u \) from the static push-over analysis.

As the displacements were then decreased, heading towards point 2, the concrete layers that were initially loaded in tension and cracked did not carry any compressive load until the cracks closed. This occurs at point 2 where \( P = -16.0 kN \text{ and } \delta = -248 \) mm. The stiffness of the frame increases noticeably at this point until the structure approaches its ultimate strength again at point 3 \( (P = -24.4 \text{ kN} \text{ and } \delta = -757 \text{ mm}) \). The average curvature at the top and bottom of each column was \( \phi = 0.182 \text{ m}^{-1} = 3.9 \phi_u \), the tension steel was well past yield
(\varepsilon_{x,avg} = 0.025 \approx 10.8 \varepsilon_{sy} \text{ and } f_{s,avg} = 474 \text{ MPa} > f_{sy} = 460 \text{ MPa}) \text{ and the concrete had crushed on the compression face (}\varepsilon_{c,avg} = 0.0055 > \varepsilon_{pc} = 0.0025). \text{ Hence, the concrete had severely crushed and cracked on both faces by point 3 and all reinforcement had gone well past the nominal yield strain.}

From point 3, the frame displacement was increased again towards point 4 (\(P = +14.8 \text{ kN and } \delta = +502 \text{ mm}\)) where the tension cracks closed and the frame stiffness increased. The frame was then moved further in the positive direction to \(\delta = +762 \text{ mm} \text{ at point 5}. \text{ The force required to hold the frame in this position (} P = 21.2 \text{ kN} \text{) was approximately 85 per cent of that required to move the frame to the same displacement that occurred during the first loading cycle (point 1). At point 5, the average curvature at the top and bottom of each column was } \phi = 0.193 \text{ m}^{-1} = 4.1 \phi_n. \text{ Furthermore, the tension steel stresses were calculated to be less (} f_s = 382 \text{ MPa} \text{) than those for point 1 even though the steel strains were greater than those for point 1 (} \varepsilon_s = 0.027 \approx 11.8 \varepsilon_{sy} \text{) as were the maximum concrete compressive strains (} \varepsilon_c = 0.0052).}

From point 5, the frame was once again pulled in the "negative" direction towards point 6 (\(P = -15.0 \text{ kN and } \delta = -528 \text{ mm}\)) where the tension cracks closed and the frame stiffened. The displacement at which this occurred was significantly greater than that at which the cracks closed on the previous cycle (point 2, \(\delta = -248 \text{ mm}\)). The force required to pull the frame further to point 7 (\(P = -21.1 \text{ kN and } \delta = -753 \text{ mm}\)) was 86 per cent of that required on the previous cycle (point 3). At point 7, the average curvature had increased to \(\phi = 0.206 \text{ m}^{-1} = 4.4 \phi_n. \text{ and the tension steel stresses were less (} f_s = 384 \text{ MPa} \text{) than those for point 3, while the strains in the steel (} \varepsilon_s = 0.029 \approx 12.6 \varepsilon_{sy} \text{) and concrete (} \varepsilon_c = 0.0056) \text{ had increased further.}

To complete the second loading cycle, the frame was then pushed from point 7 towards point 9, the tension cracks closing at point 8 (\(P = 15.1 \text{ kN and } \delta = 582 \text{ mm}\)). The force required to hold the frame in this position (point 9, \(P = 20.6 \text{ kN and } \delta = 762 \text{ mm}\)) was 97 per cent of the force for the previous cycle (point 5) and 82 per cent of the force for the 1st cycle (point 1). The average curvature at this point was \(\phi = 0.198 \text{ m}^{-1} \approx 4.2 \phi_n. \text{ the tension steel stress was the same as that at point 7 (} f_s = 384 \text{ MPa} \text{) and the strains in the steel and concrete were } \varepsilon_s = 0.028 \approx 12.2 \varepsilon_{sy} \text{ and } \varepsilon_c = 0.0052. \text{ The frame response during subsequent cycles did not change significantly from that of the 2nd loading cycle (points 5 through 9).}

In summary, the computer model predicted a considerable strength degradation, although most of this occurred during the first two cycles of loading. The shape of the load-deflection hysteresis loops was quite open ("fat"). This is in contrast
to the typical loops for moderately reinforced concrete frame elements reported by other researchers (Corvetti et al., 1993; Alaia & Griffith, 1995; Lynn et al., 1996; Bracci et al., 1995; Rodriguez and Park, 1994). In its present form, the computer model is not able to model shear effects nor joint failure. Furthermore, the spacing of stirrups in the frame under consideration would not be sufficiently close to prevent buckling of longitudinal reinforcing steel once the concrete crushes to the extent indicated by the above analysis ($\varepsilon_c = 0.0052$). The section curvatures calculated at the top and bottom of each column at the points of maximum and minimum displacement were more than four times greater than the section curvatures corresponding to the frame ultimate load from the push-over analysis. Based on these results, it appears that modifications to the program are required in order to better model the expected “pinched” hysteresis behaviour for the frame and to allow for the possibility of premature joint failure and/or shear failure of columns.

5.4 Impulse ground motion acting on slender frame

An impulse load analysis was performed on the slender frame in order to verify that the program was capable of modelling the dynamic response of the frame all the way through to final “collapse”. The frame was subjected to constant gravity load on the beam and a horizontal support movement consisting of a single cycle of a sine wave with amplitudes corresponding to a range of induced horizontal inertial forces between 0.20g to 0.357g. The horizontal displacement-time history response of the top of the column relative to the support was calculated for each of the analyses. A selection of results is shown in Fig 23. A schematic diagram of the impulse ground acceleration and displacement is shown in Fig 24. As can be seen in Figure 23, a maximum support acceleration of 0.33g was required to cause the frame to collapse. The results for this particular analysis are now discussed in detail.

The frame deformations quickly exceeded the deformation ($\delta_h = 550$ mm) observed in the static push-over analysis corresponding to the ultimate load, $P_u = 17.9$ kN. At point 1 ($t = 0.5$ sec), the relative deformation in the frame was already $\delta = 165$ mm with a base shear reaction of 13.7 kN. The corresponding displacement in the push-over analysis for the same base shear was 250 mm. No steel had yielded at this stage. By point 2 ($t = 1.0$ sec), however, the tension and middle reinforcement had yielded ($\varepsilon_{tr} = 0.0188$, $\varepsilon_{tm} = 0.0076$) at the top and bottom of each column, the compression steel had nearly yielded ($\varepsilon_{sc} = -0.0036$) and the concrete may well have crushed ($\varepsilon_c = -0.0046$). The base shear corresponding to the frame deformation of 804 mm at point 2 was 18.7 kN. The first local maximum deformation occurred at point 3 ($t = 1.5$ sec) where $\delta = 1270$ mm. At this point, the base shear reaction had dropped off to 10.5 kN
even though the steel and concrete strains had increased from the values at point 2. The frame then recovered slightly. By 3 seconds (point 4) the frame displacement had dropped back to a local minimum of 314 mm although because of the extended yielding that the steel underwent around point 3 the strains at the base of the left-hand column were all positive. Thus, at point 4 the concrete was not carrying any of the load. The base shear force in the structure at this point was -14.8 kN.

From point 4 onwards, the frame displacement continuously increased to collapse. At all points from point 2 onwards, the stress in the steel reinforcement never reached 460 MPa again even though the steel strains were well in excess of the yield strain level and the displacements were far in excess of those observed in the push-over and push-pull analyses. Somewhere between point 5 (t = 5.0 sec, δ = 1380 mm) and point 6 (t = 6.5 sec, δ = 2230 mm) the strength was lost with the base shear for these two points being 10.1 kN and 5.1 kN, respectively. From point 6 on, the frame moved rapidly to complete collapse.

In summary, a frame sidesway mechanism formed early on in the impulse analysis at lateral drifts comparable to the failure drifts observed in the push-over analysis. Once the mechanism formed, the displacements became large without an increase in base shear. Of primary importance here was the fact that the computer program was numerically stable and capable of modeling the full range of dynamic structural response through to collapse.

5.5 El Centro ground motion acting on slender frame

In this analysis, the slender frame with gravity loading was subjected to the 1940 El Centro earthquake ground motion (peak ground acceleration = 0.35g). The intent of this analysis was to establish the amplitude of lateral deformations which might be expected to occur. Keeping in mind that the slender frame had a fundamental period of T = 2.11 sec., it was apparent that the El Centro earthquake ground motion would not be particularly effective in causing distress to the frame.

The horizontal displacement time history response at the top of the column for this analysis is presented in Figure 25 for the first 10 seconds of response during which the maximum frame deformation occurred. The maximum and minimum displacements were δ = 266 mm and δ = -252 mm, respectively, both of which are well within the range of the maximum displacement observed from the static push-over analysis (δ = 550 mm). No appreciable permanent inelastic offset was observed from this analysis.
5.6 Stress-strain history for individual concrete and steel fibres in slender frame

In simulating the behaviour of the frames under any specific loading history, the stress-strain history of each fibre in each element in the frame is generated. To illustrate some typical stress-strain histories, results are presented for stress fibres in the uppermost element in the right column of the slender frame, i.e. element 76 in Fig 20-a. Attention is concentrated on the mid-point section in this element, and specifically on the outermost top and bottom steel fibres and the top and bottom fibres of concrete in this section. Typical stress-strain histories are shown in Figs 26 to 34 for simulations of pushover loading, static cyclic loading and impulse loading to collapse. As might be expected for the static loading to collapse, the stress-strain histories for the steel (Fig 26) and concrete (Fig 27) reflect the shape of the stress-strain relations of the materials. In the case of the repeated cycles of static load, the cyclic stress-strain history for the top reinforcement, shown in Fig 28, is in accordance with the constitutive relation for steel as used in the simulation program.

Fig 29 shows the stress-strain history for the top fibre concrete, which is in tension in the initial load cycle. In the first load cycle a maximum tensile strain of 0.01861 is reached at which stage the tensile stress is effectively zero. The tensile strain then decreases to zero, after which there is a build up in compressive stress as the load reverses. The peak compressive stress is reached (Fig 30, Point 3), after which the fibre softens with increasing strain. Up to this stage in the simulation, the stress-strain history is a close representation of the virgin loading curves for the concrete in tension and compression. At Point 4 the peak ‘pull’ load is reached. Unloading in compression then occurs in the fibre to Point 5. Lack of smoothness in the unloading lines in Fig 30 is due to the levels of tolerances chosen in these calculations. With the application of the second ‘push’ load the strain goes into tension, but the tensile stress remains effectively at zero. In further push-pull load cycles the stress-strain history follows the path for the first cycle, with unloading in compression again occurring at point 4.

Figs 31 and 32 show the stress-strain history in the reinforcement during impulse loading to failure. The reinforcement initially goes into tension and yields, before reversing into compression to a maximum compressive stress of about 350 MPa (i.e. not to yield). Further yielding in tension occurs before a second partial unloading to just under 400 MPa tension, as shown in Fig 31. Fig 31 gives detailed information on the first 1651 computational steps, while Fig 32 shows the full stress-strain history up to collapse.

Fig 33 shows the stress-strain history for the top concrete fibre for the first 1693 computational steps (i.e. time increments). The unloading cycles occur as in the case of the top steel reinforcement, but large strains then occur which lead, in effect, to crumbling of the concrete fibre in compression, and reduction of the stress to zero. Fig 34 shows the full range of the stress-strain history for the concrete fibre. In Figs 31 and 32 the stress-strain history for the steel shows a
very high compressive stress, in excess of the first yield stress as the frame approaches collapse. On the other hand, the surrounding concrete fibres have crumbled and in reality would provide no lateral support for the compressed reinforcing bars. Clearly the modelling of local behaviour becomes inadequate at this stage, because the steel reinforcing bars would buckle and this would lead to earlier collapse of the frame. The unrealistically high values of steel stress give corresponding high values of nodal forces and displacements (Fig 23). Not surprisingly, these results suggest that the behaviour of the slender frame is very sensitive to the magnitude of the permanent vertical load $F$. Simple collapse load calculations, made as an order-of-magnitude check, showed reasonable consistency with the results of the computer simulations. The relatively large deflections in the slender frame were also expected (Joshi et al., 1997).

### 5.7 Stocky frame analyses

The four loading cases already discussed for the slender frame were also applied to the "stocky" frame, i.e. a static push-over load, repeated static push-pull loading, an impulse ground motion and an El Centro earthquake ground motion.

The results of the static push-over analysis are presented in Fig 35 where the applied load versus horizontal displacement at the top of the left-hand column are plotted. As can be seen, the results were similar to those for the slender frame except that, as expected, $P - \delta$ effects were not significant so that there was little difference between the behaviour of the frame with and without gravity load. The ultimate strength of the stocky frame with gravity load was $P_u = 194$ kN which occurred at a displacement of $\delta_u = 27$ mm. There was no substantial yield plateau exhibited by the load-deflection plot for the stocky frame with the frame strength decreasing gradually from $P_u = 194$ kN at $\delta_u = 27$ mm to $P_u = 176$ kN at $\delta_u = 50$ mm.

The results of the static push-pull analysis were also similar to those for the slender frame. The force-displacement hysteresis curve (Fig 36) is relatively fat and there is a significant increase in stiffness due to tension cracks closing in the concrete at displacements of approximately $\delta = \pm 20$ mm. The degradation in strength over the first two cycles was 23 per cent (between points 2 and 6) in the positive direction but only 9 per cent (between points 4 and 8) in the negative direction. The average of these two values compares well with the 17 per cent degradation observed in the results for the slender frame. As for the slender frame, the steel was well past its yield stress at points 2, 4, 6 and 8 and the concrete stresses were on the softening part of their stress-strain relationship at the same points.
The results of the impulse load analysis are shown in Fig 37 where the frame displacement is plotted as a function of time for two separate analyses. The first (Impulse Ground Motion, $\Delta = 1725$ mm) analysis did not result in collapse of the frame although it resulted in a large permanent offset ($\delta = 1267$ mm) after a maximum displacement of $\delta = 1725$ mm. The second analysis ($\Delta = 1740$ mm) resulted in collapse. These large offsets and frame displacements are unrealistic for the reasons already outlined in the discussion of the slender frame. Nevertheless, the program was shown to be capable of tracking the dynamic response all the way through to collapse and this was the main objective of this analysis.

Finally, the stocky frame was subjected to the El Centro earthquake ground motion (PGA = 0.35g). The fundamental period of the stocky frame was $T = 0.14$ sec. Consequently, the earthquake input was not expected to cause very large deformations nor collapse. As can be seen from the displacement versus time plot in Fig 38 for this analysis, the maximum frame displacements were approximately 5 mm in both the positive and negative directions. In terms of material stresses and strains, the structure’s response was essentially linear and the lateral drift (in terms of the storey height) was only 0.3 per cent. The maximum base shear reaction for this analysis was 99 kN (approximately 50 per cent of the static push-over strength).

5.8 Discussion and interpretation of results

The results show that the computer-based analytical methodology is capable of modelling the dynamic response of a concrete frame up to and through yield and all the way to complete collapse. For the slender frame, this occurred with a single sine wave support acceleration of 0.33g magnitude and a period of 2.11 seconds. The corresponding drift in the single storey frame at the point where it became unstable was nearly 20 per cent of the height. This amount of drift is significantly greater than that normally associated with the ultimate limit state for a structure subject to earthquake ground motion (typically 1.5 per cent to 3 per cent drift - see Lynn et al., 1996; Bracci et al., 1995; Rodriguez and Park, 1994). Based on these results, it could be concluded that the programme is overly optimistic in its predictions of seismic strength for the concrete frames considered. The explanation for this is that the program in its present state is not able to accurately model hinge zone effects such as softening and degradation behaviour nor other potential brittle failure mechanisms such as steel fracture or buckling, or shear failure of columns or joints. As an example, the yield curvature for the 200x200 mm column with 8 Y16 bars was estimated roughly to be $\phi_y = \frac{\varepsilon_{cu} + \varepsilon_{sy}}{D} = 0.005 = 0.0025$ m$^{-1}$. (More detailed calculations assuming that the “yield curvature” corresponded to first yield of the extreme
layer of tension steel gave $\phi_y = 0.0021 \text{m}^{-1}$.) The corresponding column hinge zone (plastification region) curvature can be estimated assuming that the lateral drift $\delta = 0.14 \ h$ is due exclusively to plastic hinge rotation, $\theta_p (= \phi_{avg} \cdot L_p)$, where $\phi_{avg}$ is the average section curvature over the length of the plastic hinge $L_p$ and $h$ is the height of the frame. If it is assumed that $L_p = D = 0.2 \ \text{m}$ and the lateral drift is taken to be $0.14 \ h$, then $\theta_p = \delta h = 0.14$ and the average curvature at the point where the storey drift $\delta = 0.14 \ h$ is $\phi_p = 0.14 / 0.2 \ \text{m} = 0.7 \ \text{m}^{-1}$. The curvature ductility for the section is then estimated as $\phi_p / \phi_y = 0.7 / 0.025 = 28$ (or more accurately $f_p / f_y = 0.7 / 0.021 = 33$). This value is very high, especially since the maximum drift was actually closer to $0.2 \ h$. Of course, not all the drift is due to plastic rotation in the hinge zones. A considerable amount of drift would be expected to be due to "elastic" flexure of the slender columns. Nevertheless, even if half of the drift were due to non-hinge effects, the required section curvature ductility is still extremely high.

The implications of this are very significant. The sequence of hinge formation leading to collapse may require unachievably large ductility in the plastification regions where hinges form early on in this process. It is unlikely that modestly ductile frames can be detailed sufficiently well to allow the "early" hinges to maintain their moment capacity ($M_p$) through the very large curvatures in order to allow the last hinges to form and create a collapse mechanism. For the given portal frame the beam moment capacity $M_{pb}$ is greater than column moment capacity $M_{pc}$. First the frame was loaded with gravity loads. Thereafter the frame was loaded with horizontal push-over load. According to the moment capacities of the members the first hinge is supposed to form in one of the columns. When the horizontal push-over load is applied from left to right after the application of the gravity loads the first hinge obviously would form at the top of the right column. With a hinge formed at the top, the stiffness of the right column is only a quarter of the stiffness of the left column. The two columns now attract the induced shear force due to the increase in push-over force in the ratio of their stiffnesses. With the decrease in stiffness of the frame due to the hinge at the top of the right column, the next hinge should form at the bottom of the right column. If the push-over force is increased further, with hinges both at the top and bottom of the right column, the left column must carry all the shear force due to the increase in the push-over force. The next hinge should then form at the bottom of the left column. The fourth hinge would form at the top of the left column to create a collapse mechanism. This is what in fact is predicted by the computer analysis.

In gravity load dominated frames which have limited ductility it is highly likely that hinges which form early on in the sequence will soften, so that the moment
falls away before all the necessary hinges form to create a collapse mechanism. This could well result in either a local failure (e.g. due to instability or fracture of longitudinal reinforcement) or rapid load redistribution, either of which would reduce the load capacity of the frame. Thus, it is important to be able to model the full range of stress-strain and moment-curvature behaviour up to and including tension and compression failure of the reinforcing steel. As this is critical mainly in the regions of maximum bending moment, special “plastic-hinge zone” elements need to be developed which are capable of modelling the full range of behaviour for a concrete hinge region subject to high bending moment and including shear effects.

Of course, the load capacity may be limited by premature joint failure. Unfortunately, joint failures are commonly observed during moderate to large earthquakes, even in regions of the world with comparatively rigorous earthquake design requirements (Park et al., 1995; Holmes and Somers, 1996; Benuska, 1990). Recent Australian research has shown that joint failures can also be expected in many of our existing buildings should they be subject to a large earthquake (Corvetti et al.; 1993; Alaia et al., 1995). Hence, joint softening and degradation is a further complicating matter which has to be modelled.

5.9 Evaluation of analysis method and associated computer simulation program

The numerical study has indicated a number of strengths and weaknesses of the analysis method used. With regard first to collapse load evaluation, the analytical method and the associated computer programs have been shown to:

- be numerically stable, even when the structure is in a state of extreme overload heading towards collapse;
- give estimates of strength which correspond well with results of ultimate limit state strength calculations for the column cross-sections using the rectangular stress block theory;
- model the flexural behaviour of concrete elements over a large range of strain; and
- account for material nonlinearities as well as geometric nonlinearities.

The main short-comings of the current analysis and simulation program are:

- The program does not presently account for local buckling of the longitudinal reinforcing steel in regions where the concrete crushes.
- The analytical model underestimates the strength degradation which may be expected to occur.
• Shear failure of the columns is not presently modelled. From a design point of view this is not always a problem since designers try to avoid this type of failure. However, there continue to be many examples of shear failures of concrete columns in buildings during earthquakes that indicate that there are many existing buildings where shear failure is highly likely. It is necessary to be able to model shear failure in order to prescribe design limits to prevent this type of failure and to make realistic assessments of the seismic strength of existing buildings.

• Beam-column joint failure is not presently treated. As for shear failure of columns, this is a common failure mode for buildings in earthquakes, and a joint model needs to be developed.

6. Further work

As a result of the present study, further work is presently being undertaken in a number of different areas. Brief details are given here.

6.1 Experimental program

One of the difficulties encountered in the development of comprehensive analytic methods of dynamic analysis of structural systems is the acquisition of adequate and reliable experimental data to test the accuracy of the theoretical predictions. A body of experimental data is progressively becoming available from the pseudo-dynamic testing of large scale and prototype structural systems which is currently being undertaken internationally (Kunnath et al., 1990; Megget and Fenwick, 1989; Schultz, 1990). A series of papers was published on damaged instrumented buildings after the 1994 Northridge earthquake in Earthquake Spectra in the years 1997/98 (e.g. Li and Jirsa, 1998). While such tests can be used to check certain aspects of the theoretical analysis which relate to static behaviour they do not provide a completely satisfactory means of testing the predictive capabilities in relation to real dynamic behaviour (Kunnath et al., 1990; Megget and Fenwick, 1989). A fundamental problem in obtaining true dynamic experimental data on reinforced concrete structural systems lies in the fact that-large scale dynamic tests require very large capacity, and hence very expensive, testing facilities.

To supplement the available published test data, a number of small-scale dynamic tests are presently being undertaken which do not use reinforced concrete structures, but steel and aluminium frames with specially designed hinge regions which can be placed at any desired location. The structural models are being tested under both static and dynamic loads. The test program is
intended to parallel the analytic studies described in Section 3 of this report. The
dynamic tests are being conducted in a shaking table test facility.

The details of the frames are being chosen to ensure that both geometric and
material non-linearities affect the overload behaviour and collapse under static
and dynamic loads. The beam and column component members are relatively
slender in order to induce geometric effects, while for the hinge regions the
materials and cross-sectional configurations are chosen to provide a range of
behaviour patterns ranging from ductile, to post-peak softening, and to brittle
behaviour.

6.2 Modelling of high-moment plastification regions

As already noted, the numerical analysis described in Section 2 does not
adequately take account of degradation of high-moment regions which are
subjected to reversed cycles of high moment. In particular, buckling of the
longitudinal reinforcing steel has not been modelled for the case when large
concrete strains occur and the outer layer spalls and the interior concrete
crumbles. To deal with this important situation a special module is being
developed to allow for progressive degradation and collapse by concrete
crumbling and reinforcement buckling.

6.3 Modelling of high-moment high-shear plastification regions

The present analysis considers only flexural deformations in the beam and
column elements and therefore does not treat the possibility of shear failure.
While shear deformations can be introduced into the elements in a fairly simple
manner, a special module is needed to deal adequately with shear failure and
moment-shear failure in columns and beams under repeated cycles of loading.
The development of such a module is presently being undertaken.

6.4 Modelling of beam-column joint regions

In the test of the original prototype frame with details similar to the stocky
frame analysed in Section 3 (Alaia and Griffith, 1997), collapse actually
occurred because of premature failure in the beam-column joints. This is a
common occurrence which has not been considered to date in the analysis.
Again, a special module is being developed to treat the large deformations and
reduced moment and shear capacity which commonly occur in practice if the
detailing of the joint is not very carefully carried out.
7. Concluding remarks

In this report details have been given of a method for analysing the dynamic behaviour of concrete frames when subjected to the severe ground motions which can occur during an earthquake. The analytic method has been used as the basis for a comprehensive computer simulation procedure which can predict the behaviour of frames under both static and dynamic loads up to collapse. A large number of frame simulations has been undertaken in order to test the robustness and the adequacy of the method. The computational procedures appear to be robust and stable and in very few circumstances were convergence problems experienced.

The simulation studies have shown that some of the predictions give an over-optimistic evaluation of load capacity. The reasons for this are clear. To date, only flexural deformations have been dealt with in the theory and in the simulation programs. The possibility of premature failure in local high-shear regions in the columns has thus not been allowed for. Another deficiency occurs in pure moment regions at very large deformations, where spalling of the outer layers of concrete and crumbling of the inner core can lead to buckling of the reinforcing bars. This has not yet been taken into account. Premature failure of the joints has also been ignored.

These deficiencies are presently being corrected by the development of special modules which will be introduced into the high-moment, high-shear and joint regions in the frame to be analysed.

One of the difficulties faced in the development of complex simulation programs such as the one described here is the lack of adequate dynamic test data to check the accuracy of the predictions. The lack of data applies to concrete structures in particular because of the need for large sized test frames and the lack of very large-scale shaking table facilities.

A test program is currently being carried out which uses small slender frames made of aluminium and steel which contain special hinge regions in which strength and ductility can be adjusted. The generality of the simulation program is such that it can be applied to these frames. While such tests cannot provide the ideal check on computational accuracy, they should provide added evidence of the adequacy and adaptability of the computational method.
8. Acknowledgements

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9. References


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Appendix A: Notation

$A_j$ sectional area of the $j$-th stress fibre

$a_1$, $a_2$, $a_3$ constant for displacement functions

$B$ matrix defining relation between generalised section stresses and end forces on an element

$C$ damping matrix

$b$ ratio of strain hardening modulus to Young’s modulus in steel

$D$ vector of nodal displacements

$D_e$ vector of end displacements of element on space-fixed coordinate system

$D_{en}$ component of $D_e$

$D_k$ tangent stiffness matrix for section

$d$ vector of end displacements of an element

$d$ distance between a stress fibre and the nearest exposed surface

$E_c$ Young’s modulus of concrete

$E_{ct}$ elastic modulus of concrete characterizing incremental static deformation

$E_{hard}$ tangent modulus in strain hardening range of steel

$E_s$ Young’s modulus of steel

$E_{sc}$ secant stiffness for compressive concrete

$E_{st}$ secant stiffness for tensile concrete

$E_t$ tangent modulus for a stress fibre

$e$ eccentricity of compression load

$F_{\theta}$ vector of nodal loads

$F_e$ vector of end forces of element on space-fixed coordinate system

$f$ vector of end forces of element on moving coordinate system

$f_c$ compressive strength of concrete

$f_t$ tensile strength of concrete

$G$ vector of functions relating generalised section strains to end displacements of an element

$K_t$ tangent stiffness matrix

$K$ global tangent stiffness matrix. (The subscript $t$ is omitted for the global tangent stiffness matrix)

$K_a$ stiffness matrix of element with nonlinear terms for initial deflection

$K_g$ geometrical stiffness matrix of element

$K_{gr}$ geometrical stiffness matrix to account for rigid motion of element

$L'$ length of member

$l$ length of element

$M$ bending moment in element

$M_a$, $M_b$ end moments of element on space-fixed coordinate system

$m_{ea}$, $m_{eb}$ end moments of element on moving coordinate system

$N$ axial force in element
\( N_a, N_b \) end forces along longitudinal axis on space-fixed coordinate system
\( n_a \) end force along longitudinal axis on moving coordinate system
\( O \) vector of unbalanced nodal forces
\( P \) compression load
\( Q_a, Q_b \) end forces perpendicular to member axis on space-fixed coordinate system
\( R \) rotation angle of moving coordinate system
\( R_{np} \) parameter for stress-strain relation for steel
\( r_{ce}, r_{ct} \) constants for stress-strain relation for concrete
\( s \) vector of generalised stresses (stress resultants) at a section
\( T \) matrix for transforming from local to global coordinate systems
\( T_e \) transformation matrix for moving coordinate system
\( T_{emn} \) component of \( T_e \)
\( t \) time
\( U \) vector of functions to produce end displacements of an element from nodal displacements
\( U_a, U_b \) end displacements of element on space-fixed coordinate system
\( U_m \) component of vector \( U \)
\( u \) longitudinal displacement of element axis
\( u_e \) end displacements of element on moving coordinate system
\( V \) volume of continuum
\( V_a, V_b \) end displacements of element on space-fixed coordinate system
\( v \) lateral displacement of element axis
\( y \) nodal displacement vector
\( \Delta y \) vector of increments in nodal displacements
\( y \) distance from gravity centre of section
\( \Delta D \) increment of \( D \)
\( \Delta D_e \) increment of \( D_e \)
\( \Delta d \) increment of \( d \)
\( \Delta e \) increment of \( e \)
\( \Delta F \) increment of \( F \)
\( \Delta F_e \) increment of \( F_e \)
\( \Delta f \) increment of \( F_e \)
\( \Delta M \) increment of \( M \)
\( \Delta M_a, \Delta M_b \) increment of \( M_a, M_b \)
\( \Delta N \) increment of \( N \)
\( \Delta N_a, \Delta N_b \) increment of \( N_a, N_b \)
\( \Delta Q_a, \Delta Q_b \) increment of \( Q_a, Q_b \)
\( \Delta s \) increment of \( s \)
\( \Delta T_e \) increment of \( T_e \)
\( \Delta t \) increment of \( t \)
\( \beta \) parameter in Newmark’s method
\( \Delta u_e \) increment of \( u_e \)
\( \Delta \varepsilon \) increment of \( \varepsilon \)
\( \Delta \varepsilon_0 \) increment of \( \varepsilon_0 \)
$\Delta \varepsilon_{cr}$ increment of $\varepsilon_{cr}$
$\Delta \varepsilon_{ep}$ increment of $\varepsilon_{ep}$
$\Delta \varepsilon_{sh}$ increment of $\varepsilon_{sh}$
$\Delta \varepsilon_{sh0}$ increment of $\varepsilon_{sh0}$
$\Delta \kappa$ increment of $\kappa$
$\Delta \lambda$ increment of $\lambda$
$\Delta \sigma$ increment of $\sigma$
$d$ displacement
$\delta_1, \delta_2, \delta_3$ displacements of RC frame
$\delta d$ virtual component of $d$
$\delta e$ virtual component of $e$
$\delta u$ virtual component of $u$
$\delta u_e$ virtual component of $u_e$
$\delta v$ virtual component of $v$
$\delta \varepsilon$ virtual component of $\varepsilon$
$\delta \varepsilon_0$ virtual component of $\varepsilon_0$
$\delta \kappa$ virtual component of $\kappa$
$\delta \theta_{ea}$ virtual component of $\theta_{ea}$
$\delta \theta_{eb}$ virtual component of $\theta_{eb}$
$\varepsilon$ total strain in stress fibre
$\varepsilon_0$ total strain at gravity center of a section
$\varepsilon_{pc}$ strain at compressive peak stress in stress-strain relation model for concrete
$\varepsilon_{pt}$ strain at tensile peak stress in stress-strain relation model for concrete
$\varepsilon_y$ yield strain in steel ($=\sigma_y/E_y$)
$\kappa$ curvature of element
$\lambda$ load factor
$\theta_{ea}$ end displacement of element on moving coordinate system
$\theta_{eb}$ end displacement of element on moving coordinate system
$\sigma$ normal stress
$\sigma_{max}$ stress at extreme compressive face
$\sigma_{min}$ stress at extreme tensile face
$\sigma_y$ yield stress in steel
Appendix B: Mathematical details of non-linear analysis

B.1 Deformations, strains and stresses in an element in local coordinates

We first consider the deformations in a beam element in a local axis system $x$-$y$ which is fixed at end A of the element, with the $x$ axis passing along the element and through the other end B. The element is shown in Fig B1. The vector of end displacements is:

$$
\mathbf{d} = \begin{bmatrix} u_e \\ \theta_{ea} \\ \theta_{eb} \end{bmatrix}
$$

(1)

and the corresponding vector of end forces is:

$$
\mathbf{f} = \begin{bmatrix} n_e \\ m_{ea} \\ m_{eb} \end{bmatrix}
$$

(2)

The longitudinal and lateral displacements of a point on the centroid of the element, $u(x)$ and $v(x)$, can be expressed in terms of the end displacements using polynomials in $x$ as follows:

$$
u(x) = a_2 \frac{x}{l} \left[ \frac{x}{l} - 1 \right] + a_3 \frac{x}{l} \left[ \left( \frac{x}{l} \right)^2 - 1 \right]
$$

(4)

The term $l$ is the initial (undeformed) length of the bar element. At the ends, the following values apply:

$$
u(l) = a_1 \frac{x}{l} = u_e
$$

(5)

$$
\frac{d\nu(0)}{dx} = \frac{a_2}{l} - \frac{a_3}{l} = \theta_{ea}
$$

(6)

$$
\frac{d\nu(l)}{dx} = \frac{a_2}{l} + \frac{2a_3}{l} = \theta_{eb}
$$

(7)
The parameters $a_1$, $a_2$ and $a_3$ can thus be expressed in terms of the end displacements $u_e$, $\theta_{ea}$ and $\theta_{eb}$ and Eqs 3 and 4 can be rewritten as follows:

$$u(x) = \frac{x}{l} u_e$$

(8)

$$\nu(x) = l \left\{ \left[ \left( \frac{x}{l} \right)^3 - 2 \left( \frac{x}{l} \right)^2 + \frac{x}{l} \right] \theta_{ea} + \left[ \left( \frac{x}{l} \right)^3 - \left( \frac{x}{l} \right)^2 \right] \theta_{eb} \right\}$$

(9)

Differentiating with respect to $x$ we have:

$$\frac{du(x)}{dx} = \frac{u_e}{l}$$

(10)

$$\frac{d\nu(x)}{dx} = \xi_a \theta_{ea} + \xi_b \theta_{eb}$$

(11)

$$\frac{d^2 \nu(x)}{dx^2} = \frac{1}{l} \left[ 6 \left( \frac{x}{l} \right) - 4 \right] \theta_{ea} + \frac{1}{l} \left[ 6 \left( \frac{x}{l} \right) - 2 \right] \theta_{eb}$$

(12)

where $\xi_a$ and $\xi_b$ are polynomials of the coordinate $x$, i.e:

$$\xi_a = 3 \left( \frac{x}{l} \right)^2 - 4 \frac{x}{l} + 1$$

(13)

$$\xi_b = 3 \left( \frac{x}{l} \right)^2 - 2 \frac{x}{l}$$

(14)

Using Green’s strain in order to allow for second order geometric effects, the strain at the centroid of the beam element is:

$$\varepsilon_0(x) = \frac{du(x)}{dx} + \frac{1}{2} \left[ \frac{du(x)}{dx} \right]^2 + \frac{1}{2} \left[ \frac{d\nu(x)}{dx} \right]^2$$

(15)

The curvature is approximately defined as:

$$\kappa(x) = \frac{d^2 \nu(x)}{dx^2}$$

(16)

By substituting Eqs 10, 11 and 12 into Eqs 15 and 16, we can express $\varepsilon_0(x)$ and $\kappa(x)$ in terms of the end displacements. Hence, treating $\varepsilon_0(x)$ and $\kappa(x)$ as components of a generalised strain vector $\mathbf{e}(x)$, we can write:
\[ e(x) = G(x, d) \]  

(17)

where \( d \) is the vector of end displacements of the element (Eq 1), and \( G(x, d) \) is a transformation function:

\[
G(x, d) = \begin{bmatrix}
\frac{u_e}{l} + \frac{u_e^2}{2l^2} + \left[ \xi_a \theta_{ea} + \xi_b \theta_{eb} \right]^2 \\
\frac{1}{l} \left[ 6 \frac{x}{l} - 4 \right] \theta_{ea} + \frac{1}{l} \left[ 6 \frac{x}{l} - 2 \right] \theta_{eb}
\end{bmatrix}
\]  

(18)

The section is subdivided into thin layers of concrete and reinforcing steel. The stress in the \( j \)-th layer is determined from the strain \( \varepsilon_j \) by means of the relation:

\[
\sigma_j = \sigma_j(\varepsilon_j)
\]  

(19)

where \( \sigma_j(\varepsilon_j) \) represents the instantaneous stress-strain relation for this fibre. The assumption that plane sections remain plane in the section of an element leads to:

\[
\varepsilon_j = \varepsilon_0 - y_j \kappa
\]  

(20)

where \( y \) is the distance from the centroid (Fig 1).

The stress resultants are obtained by summing the forces in all layers in the section:

\[
s = \begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} \sum_j A_j \sigma_j \\ -\sum_j A_j y_j \sigma_j \end{bmatrix}
\]  

(21)

where \( N \) is the axial force, \( M \) is the bending moment, \( A_j \) is the sectional area of the \( j \)-th fibre, and \( s \) is the vector of generalised stresses.

These equations, together with appropriate constitutive relations for the concrete and steel, allow the strains, stresses and stress resultants in a beam element to be determined from a given or assumed set of end displacements \( d \) in local axes.

**B.2 Tangent stiffness relation for the cross-section of a beam element**

We now consider the tangent stiffness relations for the element in local axes. The incremental form of the stress-strain relation is expressed approximately as:
\[ \Delta \sigma_j = \frac{d \sigma_j (\varepsilon_j)}{d \varepsilon_j} \Delta \varepsilon_j = E_j \Delta \varepsilon_j \]  

(22)

Here, the tangent modulus \( E_j \) is dependent on the value of \( \varepsilon_j \). From Eq 25, the increment in the strain \( \Delta \varepsilon_j \) is:

\[ \Delta \varepsilon_j = \Delta \varepsilon_0 - y_j \Delta \kappa \]  

(23)

The increments in the stress resultants can be written as the vector \( \Delta s \):

\[ \Delta s = \begin{bmatrix} \Delta N \\ \Delta M \end{bmatrix} = \begin{bmatrix} \sum_j A_j \Delta \sigma_j \\ -\sum_j A_j y_j \Delta \sigma_j \end{bmatrix} \]  

(24)

An equation can be obtained in the following form by using Eqs 23 and 24:

\[ \Delta s = D_t \Delta e \]  

(25)

where

\[ \Delta e = \begin{bmatrix} \Delta \varepsilon_0 \\ \Delta \kappa \end{bmatrix} \]  

(26)

and

\[ D_t = \begin{bmatrix} \sum_j E_{ij} A_j & -\sum_j E_{ij} A_j y_j \\ \sum_j E_{ij} A_j y_j & \sum_j E_{ij} A_j y_j^2 \end{bmatrix} \]  

(27)

\( D_t \) is the tangent stiffness matrix, and \( \Delta e \) is the vector of increments in generalised strains. Equations 25 to 27 apply to a cross-section in the element.

**B.3 Tangent stiffness relation for a beam element in local axes**

To derive stiffness expressions for the entire element, it is convenient to use the principle of virtual work. For the beam element, the principle is expressed as follows:
where $\sigma$ is the normal stress in a section, $\delta e$ is the virtual strain, $\delta d$ is the vector of virtual end displacements (Eq 24), and $f$ is the vector of end forces (Eq 2). $V$ is the initial (undeformed) volume of the bar element, within which the integral is performed.

From Eqs 15 and 16, the virtual components of the generalised strains may be expressed as follows:

$$
\delta e = \begin{bmatrix} \delta e^x \\ \delta e^y \\ \delta e^z \end{bmatrix} = \begin{bmatrix} \frac{d\delta u}{dx} + \frac{dvd\delta v}{dx} \\ \frac{dv}{dx} \end{bmatrix} = \begin{bmatrix} \frac{d^2\delta v}{dx^2} \end{bmatrix}
$$

(29)

where the coordinate $x$ is omitted for simplicity. The differentials of virtual components $\delta u$ and $\delta v$ may be expressed as follows by referring to Eqs 10 to 12:

$$
\frac{d\delta u}{dx} = \frac{\delta u}{l}
$$

(30)

$$
\frac{d\delta v}{dx} = \xi_e \delta \theta_{ea} + \xi_e \delta \theta_{eb}
$$

(31)

$$
\frac{d^2\delta v}{dx^2} = \frac{1}{l} \left[ 6 \frac{x}{l} - 4 \right] \delta \theta_{ea} + \frac{1}{l} \left[ 6 \frac{x}{l} - 2 \right] \delta \theta_{eb}
$$

(32)

Substituting into Eq 29 we obtain

$$
\delta e = B \delta d
$$

(33)

$$
B = \begin{bmatrix}
\frac{1}{l} \left[ 1 + \frac{u}{l} \right] & \xi_e \delta \theta_{ea} + \xi_e \delta \theta_{eb} & \xi_e \xi_e \delta \theta_{ea} + \xi_e \xi_e \delta \theta_{eb} \\
0 & \frac{1}{l} \left[ 6 \frac{x}{l} - 4 \right] & \frac{1}{l} \left[ 6 \frac{x}{l} - 2 \right]
\end{bmatrix}
$$

(34)

where $B$ is the stress-displacement matrix, and $\delta d$ is the vector of virtual end displacements.

From Eqs 19, 28 and 29:
\[ \int_{V} \sigma (\delta e_{0} - y \delta \kappa) dV = \delta d' f \]  

(35)

Performing first the integration over the cross section, we can rewrite Eq 35 as:

\[ \int_{0}^{l} \delta e' s dx = \delta d' f \]  

(36)

where \( \delta e \) is the vector of virtual components of the generalised strains (Eq 24), \( s \) is the vector of generalised stresses (Eq 21), and \( l \) is the length of the bar element.

By substituting Eq 33 into Eq 36,

\[ \delta d' \int_{0}^{l} B' s dx = \delta d' f \]  

(37)

which leads to the following equation, since \( \delta d \) may have arbitrary values:

\[ f = \int_{0}^{l} B' s dx \]  

(38)

This equation represents the transformation from generalised stresses into the end forces of the element.

The increments of the generalised strains may be expressed in a similar form to Eq 33.

\[ \Delta e = B \Delta d \]  

(39)

\[ \Delta d = \begin{bmatrix} \Delta u_e \\ \Delta \theta_{ea} \\ \Delta \theta_{eb} \end{bmatrix} \]  

(40)

where \( \Delta d \) is a vector of the increments of end displacements.

The principle of virtual work can be applied to the increments. From Eq 29, the virtual strain caused by virtual displacements is:

\[ \delta \varepsilon = \frac{d \delta u}{dx} + \frac{d u d \delta u}{dx} + \frac{d v d \delta v}{dx} - y \frac{d^2 \delta v}{dx^2} \]  

(41)
As the element undergoes an increment in deformation (e.g., due to a load increment) the virtual increment in strain is:

$$\delta (\varepsilon + \Delta \varepsilon) = \frac{d \delta u}{dx} + \frac{d [u + \Delta u]}{dx} \frac{d \delta u}{dx} + \frac{d [v + \Delta v]}{dx} \frac{d \delta v}{dx} - \gamma \frac{d^2 \delta v}{dx^2}$$

$$= \delta \varepsilon + \frac{d \Delta u d \delta u}{dx} + \frac{d \Delta v d \delta v}{dx}$$

(42)

The principle of virtual work leads to the following equations for the previous state and the new state, respectively:

$$\int_V \sigma \delta \varepsilon dV = \delta d^t f$$

(43)

$$\int_V [\sigma + \Delta \sigma] \delta (\varepsilon + \Delta \varepsilon) dV = \delta d^t [f + \Delta f]$$

(44)

Subtracting Eq 43 from Eq 44,

$$\int_V \Delta \sigma \delta \varepsilon dV + \int_V [\sigma + \Delta \sigma] \left[ \frac{d \Delta u d \delta u}{dx} + \frac{d \Delta v d \delta v}{dx} \right] dV = \delta d^t \Delta f$$

(45)

By ignoring the high order terms, the above equation yields:

$$\int_V \Delta \sigma \delta \varepsilon dV + \int_V \sigma \left[ \frac{d \Delta u d \delta u}{dx} + \frac{d \Delta v d \delta v}{dx} \right] dV = \delta d^t \Delta f$$

(46)

which is a statement of the principle of virtual work for increments. The equation can also be rewritten in the following form using stress resultants:

$$\int_0^l \delta e^t d s dx + \int_0^l N \left[ \frac{d \Delta u d \delta u}{dx} + \frac{d \Delta v d \delta v}{dx} \right] dx = \delta d^t \Delta f$$

(47)

The first term in the left side of Eq 47 can be rewritten in the following form by substituting Eqs 25, 33 and 38:

$$\int_0^l \delta e^t d s dx = \delta d^t K_a \Delta d$$

(48)

Here, $K_a$ is a nonlinear function of the end displacements, and represents the effect of initial deflections (due to prior load increments):
\[
K_a = \int_{0}^{l} B'D_i B \, dx
\]  
(49)

If the end displacements are set to zero, then \( K_a \) is identical to the ordinary stiffness matrix for geometrically-linear problems.

The second term in the left side of Eq 47 comprises the so called geometrical stiffness matrix \( K_g \). The axial redundant \( N \) can be expressed in terms of the axial end force \( n_e \) and the displacement \( u_e \). To do this, we first integrate Eq 35 by parts, using Eq 29:

\[
\begin{align*}
\int_{0}^{l} \left[ d \left( N \left[ 1 + \frac{du}{dx} \right] \right) \right] \delta u \, dx &+ \int_{0}^{l} \left( \frac{d}{dx} \left[ N \frac{dv}{dx} \right] - \frac{d^2 M}{dx^2} \right) \delta v \, dx \\
&- N \left[ 1 + \frac{du}{dx} \right] \delta u \bigg|_{x=0}^{l} - \left[ N \frac{dv}{dx} - \frac{dM}{dx} \right] \delta v \bigg|_{x=0}^{l} - M \frac{d}{dx} \delta v \bigg|_{x=0}^{l} + \delta d' f = 0
\end{align*}
\]

As the above equation is valid for arbitrary virtual displacements, the following equilibrium equations apply:

\[
\begin{align*}
\frac{d}{dx} \left( N \left[ 1 + \frac{du}{dx} \right] \right) &= 0 \tag{50} \\
\frac{d}{dx} \left[ N \frac{dv}{dx} \right] - \frac{d^2 M}{dx^2} &= 0
\end{align*}
\]

The boundary conditions are:

\[
\begin{align*}
\delta u (0) &= 0 \\
\delta v (0) &= 0 \\
\delta v (l) &= 0 \\
n_e - N (l) \left[ 1 + \frac{du (l)}{dx} \right] &= 0 \\
m_{e_a} + M (0) &= 0 \\
m_{e_b} - M (l) &= 0
\end{align*}
\]

From the approximation of \( u \) (Eq 28), the differential of \( u \) is:

\[
\frac{du}{dx} = \frac{u_e}{l} \tag{51}
\]

Substituting into the first equilibrium equation,
\[ \frac{dN}{dx} = 0 \]  \hspace{1cm} (52)

This simply means that the axial force \( N \) is constant along the element length. From Eq 51 and the fourth boundary condition,

\[ N = \frac{n_e}{1 + u_e/l} \]  \hspace{1cm} (53)

The redundant \( N \) thus depends on \( u_e \) as well as \( n_e \). The same relation can also be obtained from the fundamental expressions of the second Piola-Kirchhoff’s stresses.

Substituting Eqs 15, 21 and 53 into the second term in the left side of Eq 47:

\[ \int_0^l \left[ N \left( \frac{d\Delta u d\Delta u}{dx} + \frac{d\Delta v d\Delta v}{dx} \right) \right] dx = \delta d^t K_g \Delta d \]  \hspace{1cm} (54)

\[ K_g = \frac{n_e}{1 + u_e/l} \begin{bmatrix} 1/l & 0 & 0 \\ 2l/15 & -l/30 & \text{sym.} \\ 2l/15 & \end{bmatrix} \]  \hspace{1cm} (55)

where \( K_g \) represents the second-order effect of the existing stresses, and guarantees an accurate treatment of geometrical nonlinearities, even when the discretisation in the longitudinal direction of a member is coarse. Substituting Eqs 47 and 54 into Eq 47,

\[ \delta d^t K_a \Delta d - \delta d^t K_g \Delta d = \delta d^t \Delta f \]  \hspace{1cm} (56)

Noting the arbitrary nature of \( \delta d \) we can write

\[ \Delta f = [K_a + K_g] \Delta d \]  \hspace{1cm} (57)

We thus have the tangent stiffness of the bar element in the local axis system. The component \( K_g \) takes account of the geometric non-linearity associated with bending within the length of the flexural element.
B.4 Transformation to system coordinate system and global stiffness relations

The origin of the $x$-$y$ axes is attached to the bar element at end A and moves as the global frame deforms. The other end point B remains on the $x$ axis (Fig B4). The axis system $x_0$-$y_0$ is fixed in space at the initial location of the element. The geometrical relations between the end displacements measured in the $x_0$-$y_0$ and $x$-$y$ axes are expressed as:

\[
U_b - U_a + l = (l + u_e) \cos R \\
V_b - V_a = (l + u_e) \sin R \\
\theta_a = \theta_{ea} + R \\
\theta_b = \theta_{eb} + R
\]  

(58)

or

\[
d = U(D_e)
\]  

(59)

where

\[
D_e = [U_a \ V_a \ \theta_a \ U_b \ V_b \ \theta_b]'
\]  

(60)

\[
U(D_e) = \begin{bmatrix}
\sqrt{(U_b - U_a + l)^2 - (V_b - V_a)^2 - l} \\
\theta_a - R \\
\theta_b - R
\end{bmatrix}
\]  

(61)

and

\[
R = \arctan\left(\frac{V_b - V_a}{U_b - U_a + 1}\right)
\]  

(62)

The vector of end displacements $D_e$ is measured on the space-fixed coordinate system $x_0$-$y_0$, and $U(D_e)$ represents the vector of transformation functions from $D_e$ to $d$.

The relations between the increments can be obtained from Eq 61:

\[
\Delta d = T_e \Delta D_e
\]  

\[
T_{enn} = \frac{\partial U_n}{\partial D_{en}}
\]  

(63)
where \( T_e \) is the transformation matrix, and \( T_{en} \) denotes the component \((m, n)\) in \( T_e \). \( U_m \) and \( D_n \) are also components in \( U(D_e) \) and \( D_e \), respectively. By performing the differentiation, \( T_e \) is expressed as:

\[
T_e = \begin{bmatrix}
-\cos R & -\sin R & 0 & \cos R & \sin R & 0 \\
\sin R & \cos R & 1 & \sin R & \cos R & 0 \\
l + u_e & l + u_e & 0 & l + u_e & l + u_e & 1 \\
l + u_e & l + u_e & 0 & l + u_e & l + u_e & 1 \\
\end{bmatrix}
\]

From the contragradience theorem, the transformation for the end forces yields:

\[
F_e = T_e^T f
\]

(64)

The components of the vector \( F_e \),

\[
F_e = \begin{bmatrix}
N_a \\
Q_a \\
M_a \\
N_b \\
Q_b \\
M_b \\
\end{bmatrix}
\]

are the end forces measured in \( x_0 - y_0 \). The increment \( \Delta F_e \) is derived from Eq 64:

\[
\Delta F_e = T_e^T \Delta f + \Delta T_e^T f
\]

(65)

\[
\Delta F_e = \begin{bmatrix}
\Delta N_a \\
\Delta Q_a \\
\Delta M_a \\
\Delta N_b \\
\Delta Q_b \\
\Delta M_b \\
\end{bmatrix}
\]

The second term on the right side of Eq 65 represents the rotation of the coordinate system \( x - y \), and has the form:
\[ \Delta T_e^j f = \begin{bmatrix} -\Delta (\cos R) n_e - \Delta \left( \frac{\sin R}{l + u_e} \right) (m_e + m_e b) \\ -\Delta (\sin R) n_e + \Delta \left( \frac{\cos R}{l + u_e} \right) (m_e + m_e b) \\ 0 \\ \Delta (\cos R) n_e + \Delta \left( \frac{\sin R}{l + u_e} \right) (m_e + m_e b) \\ \Delta (\sin R) n_e - \Delta \left( \frac{\cos R}{l + u_e} \right) (m_e + m_e b) \\ 0 \end{bmatrix} \]  

(66)

The incremental terms in the above equation can be rewritten as:

\[ \Delta (\ ) = \begin{bmatrix} \frac{\partial}{\partial U_a} (\ ) & \frac{\partial}{\partial V_a} (\ ) & \frac{\partial}{\partial \theta_a} (\ ) & \frac{\partial}{\partial U_b} (\ ) & \frac{\partial}{\partial V_b} (\ ) & \frac{\partial}{\partial \theta_b} (\ ) \end{bmatrix} \Delta D_e \]

By performing the above differentiations, \( \Delta T_e^j f \) is expressed as:

\[ \Delta T_e^j f = K_{gr} \Delta D_e \]  

(67)

\[ K_{gr} = \begin{bmatrix} K_a & K_b & 0 & -K_a & -K_b & 0 \\ K_c & 0 & -K_b & -K_c & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ K_a = \frac{n_e \sin^2 R - 2 q_e \sin R \cos R}{[l + u_e]} \]

\[ K_b = \frac{n_e \sin R \cos R + q_e (\sin^2 R - \cos R)}{[l + u_e]} \]

\[ K_c = \frac{n_e \cos^2 R + 2 q_e \sin R \cos R}{[l + u_e]} \]

\[ q_e = \frac{m_e + m_e b}{[l + u_e]} \]

\( K_{gr} \) is the geometrical stiffness matrix representing the second order effect due to the rigid motion of a bar element.

Substituting Eqs 57, 63 and 67 into 65, the tangent stiffness relation expressed in the space-fixed, local coordinate system \( x_0 y_0 \) is given as follows:
\[ \Delta F_e = \{ T_e^t [K_a + K_g] T_e + K_{gr} \} \Delta D_e \]  

(68)

To construct the global tangent stiffness relation, the transformation of Eq 68 is required from the space-fixed local coordinate system \( x_0'y_0 \) into the global coordinate system \( X-Y \).

\[ \Delta F = \sum_i T_i^t \Delta F_{ei} \]  

\[ D_{ei} = T_i D, \quad \Delta D_{ei} = T_i \Delta D \]  

(69)

where \( \Delta F \) is the vector of nodal load increments, \( \Delta D \) is the vector of increments of nodal displacements, and \( T_i \) is the transformation matrix from \( x_0'y_0 \) into \( X-Y \). The subscript \( i \) denotes the \( i \)-th bar element. From Eqs 68 and 69, the global tangent stiffness relation is derived as follows:

\[ \Delta F = K \Delta D \]  

\[ K = \sum_i T_i^t \{ T_{ei}^t [K_{ai} + K_{gi}] T_{ei} + K_{gi} \} T_i \]  

(70)

\( K \) is the global tangent stiffness matrix, \( \Delta D \) is the vector of increments of nodal displacements, and the subscript \( i \) denotes the \( i \)-th bar element.

Eq 70 has the form:

\[ K = K_a + K_g + K_{gr} \]  

(71)

in which \( K_g \) and \( K_{gr} \) take account of the geometric effects. In particular, \( K_g \) allows for the overall displacement of the element during the previous load increments, while \( K_{gr} \) allows for bending displacements within the length of the element.
Fig 1. Frame on rigid moving base

Fig 2. Mass on moving base
\[ y_n = y_{n-1} + \Delta y_n \]
\[ y_n = y_{n-1} + \delta y_n \]

Fig 3. Newton-Raphson method: Step in calculation
Fig 4. Modified Popovics, model for confined concrete

Fig 5. Menegotto & Pinto model for steel
Fig 6. RC frame

Fig 7. Columns and beams

Fig 8. Vertical loads (kN)

Fig 9. Concentrated masses ($10^3$kg)
Fig 10. Definition of nodes

Fig 11. Definition of beam elements

Fig 12. Definition of column elements
<table>
<thead>
<tr>
<th>Order</th>
<th>Eigen values</th>
<th>Natural periods</th>
<th>Circular frequencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.19681E+03</td>
<td>0.44787E+00</td>
<td>0.14029E+02</td>
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<tr>
<td>2</td>
<td>0.15412E+04</td>
<td>0.16005E+00</td>
<td>0.39258E+02</td>
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<tr>
<td>3</td>
<td>0.39116E+04</td>
<td>0.10046E+00</td>
<td>0.62542E+02</td>
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<td>0.83928E+04</td>
<td>0.68584E-01</td>
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<td>0.15220E+05</td>
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<td>0.12337E+03</td>
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<td>6</td>
<td>0.27371E+05</td>
<td>0.37978E-01</td>
<td>0.16544E+03</td>
</tr>
</tbody>
</table>

Fig 13. Modes of motion (the 1st to the 6th)
Fig 14. Response of floor level displacements
(El Centro 1940 N-S, maximum acceleration = 3410 mm s\(^{-2}\))

Fig 15. Response of floor level velocities
(El Centro 1940 N-S, maximum acceleration = 3410 mm s\(^{-2}\))

Fig 16. Response of floor level accelerations
(El Centro 1940 N-S, maximum acceleration = 3410 mm s\(^{-2}\))
Fig 17. Base shear
(El Centro 1940 N-S, maximum acceleration = 3410 mm s\(^{-2}\))

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Fig 19. Energy Balance
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Fig 20. Portal frame for analysis
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Fig 22. Force Displacement Curve for left beam column joint in the slender frame under 8 cyclic loading of 1.5 $\delta_{ult}$ horizontal load at the Joint
Slender Frame with 26 $F$, each of $F = 5.184$ kN,
Impulse ground Motion (in resonance), $t_1 = 2.11$ s

Fig 23. Response displacement versus time for left beam-column joint
Fig 24. Ground Motions
Slender Frame with 26 of $F$, each $F = 5.184$ kN, El Centro Ground Motion, $a_g$

Fig 25. Response Displacement versus Time for Left Beam-Column Joint

$\delta_{\text{max}} = -252.31 \text{ mm, } t = 4.92 \text{ s}$

$\delta_{\text{max}} = 266.26 \text{ mm, } t = 9.44 \text{ s}$
Slender frame, top element of right column, intermediate section; all reinforcements

Fig 26. Stress-strain history of steel for push-over loading
Fig 27. Strain-strain history of concrete for push-over loading.
Fig 28. Stress-strain history of steel for cyclic loading.
Figure 29. Stress-strain history of concrete for cyclic loading (first stage).

Stress, strain element of initial column, intermediate section, top concrete.
Fig 30. Stress-strain history of concrete for cyclic loading

Slender frame, top element of right column, intermediate section; top concrete
Slender frame, top element of right column, intermediate section; top reinforcements (first 1651 increments)

Fig 31. Stress-strain history of steel for impulse loading (first stage)
Slender frame, top element of right column, intermediate section; top reinforcements (all increments except the last one)

Fig 32. Stress-strain history of steel for impulse loading
Slender frame, top element of right column, intermediate section; top concrete (first 1693 increments)

Fig 33. Stress-strain history of concrete for impulse loading (first stage)
1: $P_u = 195.73$ kN, $\delta_u = 29.0$ mm, with only horizontal pushover force $P$ at left beam-column joint (inc. =10).

2: $P_u = 193.58$ kN, $\delta_u = 27.45$ mm, with horizontal pushover force $P$ at left beam-column joint and gravity load of beam & slab, 26 $F$, each of $F = 5.184$ kN (inc. =13).

Fig 35. Force Displacement curve for left beam column joint in the stocky frame under pushover loading of horizontal load at the Joint.
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Stocky Frame, 26 equal $F$'s each of magnitude 5.184 kN,
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Fig 38. Response Displacement versus Time for left beam-column joint
Fig B1. Displacements of beam elements

Fig B2. End forces and stress resultants of beam element

Fig B3. Discretisation of section
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