A new approach for interfacial stress analysis of beams bonded with a thin plate

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**ABSTRACT**

Extensive research has shown that bonding a fibre-reinforced polymer (FRP) plate to the tension face of a reinforced concrete (RC) or steel beam can effectively enhance its serviceability and ultimate strength. The controlling failure mode of such a strengthened beam often involves the premature debonding of the FRP plate from the original beam in a brittle manner. A solid understanding of the cause and mechanism of this debonding failure mode is important for the development of an accurate strength model so that this strengthening technique can be used more effectively and economically. This paper presents a new analytical solution for the interfacial stresses in simply supported beams bonded with a thin plate and subjected to arbitrary symmetric loads. The solution is represented by Fourier series and is based on the minimisation of the complementary energy. It not only takes into consideration the non-uniform stress distribution in the adhesive layer and the stress-free boundary condition at the ends of the plate, but also correctly predicts the drastic difference in the interfacial normal stress between the plate-to-adhesive interface and adhesive-to-concrete interface.

**KEYWORDS**

FRP, RC beams, strengthening, interfacial stresses, analytical solution.

**Introduction**

Extensive research has shown that bonding a fibre-reinforced polymer (FRP) plate to the tension face (or the soffit in the context of a simply supported beam) of a reinforced concrete (RC) beam can effectively enhance its serviceability and ultimate strength (Teng et al. 2002a). Research has shown that the controlling failure mode of such a strengthened beam often involves the premature debonding of the FRP plate from the beam in a brittle manner (Smith and Teng 2001a, 2001b, 2003). Extensive studies have been carried out during the last decade on the prediction of interfacial stresses, generally within the context E of RC beams strengthened with an FRP plate, although a substantial amount of work on interfacial stresses in steel plated RC beams had been carried out before FRP plate bonding became popular. These include experimental studies (e.g. Garden et al. 1998; Ahmed et al. 2001; Bonacci and Maalej 2001), numerical studies using the linear finite element method (e.g. Täljsten 1997; Malek et al. 1998; Rabinovich and Frostig 2000; Teng et al. 2002b) and the nonlinear finite element method (e.g. Ascione and Feo 2000; Rahimi and Hutchinson 2001; Aprile et al. 2001), discrete section analysis (e.g. Arduini and Nanni 1997) and analytical solutions (e.g. Smith and Teng 2001).

**METHOD OF SOLUTION**

**Geometry and Loading**

Consider a simply supported RC beam with a span of 2*L*. The bonded plate has a length of 2*l* (Figure 1). The beam is subjected to an axial force *N*0, a pair of end moments *M*0 and a symmetrically distributed arbitrary transverse load *q*(*x*). It may be noted that any thermal loading due to the difference in thermal properties between the materials for the beam and the plate (e.g. FRP, concrete, cast iron) can be easily included in *N0*.



Figure 1. A plated beam under symmetric loads

**Assumptions**

The present analysis takes into consideration the transverse shear stress and strain in the RC beam and the FRP plate but ignores the transverse normal stress in them. Additionally, the following four assumptions are adopted:

(1) each individual layer is elastic, homogeneous and orthotropic. Note that the assumption of orthotropic behaviour has implications only for the shear moduli of the materials for the RC beam and the bonded plate;

(2) the three layers are perfectly bonded (no slips or opening-up at the interfaces);

(3) the Euler-Bernoulli beam theory is adopted for the beam and the plate, whereas the adhesive layer is considered to be in a plane stress state; and

(4) the longitudinal stress in the adhesive is assumed to vary linearly across its thickness.

**Equilibrium Equations of Beam and Plate**

For the beam and plate (*i*th layer, *i* = 1, 3), equilibrium considerations lead to the following relations

 (1a)

 (1b)

 (1c)

where ,  and are the axial force, shear force and bending moment respectively in the *ith* layer and  and  are the shear and transverse normal stresses respectively at the *ith* interface. In Eq. 1 and the rest of this paper, the superscript in *x*[*i*] is omitted because the global and the three local co-ordinate systems share the same horizontal axis.

**Representation of Stress Fields**

*Stress field in the adhesive layer*

The adhesive layer is treated as an elastic continuum without any body force. The equilibrium equations in its local coordinate system are

 (2)

 (3)

where ,  and  denote the longitudinal, shear and transverse stresses respectively. The equilibrium conditions of Eqs. 2 and 3 lead to other equations.

*Stress fields in the plate and the RC beam*

Using Assumption 3, the longitudinal and shear stresses in the plate and the beam can be expressed as

 (4a)

 (4b)

Eqs. 4a and 4b form the basis of the solution.

**RESULTS AND DISCUSSION**

In Figure 2, the bond strengths predicted using the proposed bond-slip models are compared with the results of the 253 pull tests in Lu et al.’s (2005) database. It can be seen that the proposed bond-slip models give results in close agreement with the test results and perform better than any other bond-slip models. The results of the precise model and the simplified model are almost the same, with the precise model performing very slightly better. Table 1 shows that the prediction of the proposed bi-linear model for the bond strength, which can be given as a closed-form expression (Lu et al. 2005), performs significantly better than all existing bond strength models except Chen and Teng’s (2001) model. For the prediction of bond strength, Chen and Teng’s (2001) model is still recommended for use in design due to its simple form and good accuracy.



(a) Precise model (b) Bilinear model

Figure 2. Bond strengths: test results versus predictions of proposed bond-slip models

Table 1. Predicted-to-test bond strength ratios: bond strength models

|  |  |  |  |
| --- | --- | --- | --- |
| Bond strength model | Average Predicted-to-test bond strength ratio | Coefficient of variation | Correlation coefficient |
| Chaallal *et al.* (1998) | 1.683 | 0.749 | 0.240 |
| Khalifa *et al.* (1998) | 0.680 | 0.293 | 0.794 |
| Chen and Teng (2001) | 1.001 | 0.163 | 0.903 |
| Proposed, bilinear model | 1.001 | 0.156 | 0.908 |

**CONCLUSIONS**

This paper has presented materials extracted from existing papers to illustrate the style requirements of papers to be submitted for publication in the proceedings of the Ninth Asia-Pacific Conference on FRP in Structures to be held in Adelaide, Australia, on 8-11 December 2024.

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